Large-Type Edition

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, August 17, 2023 — 12:30 to 3:30 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 A plane intersects a sphere. Which two-dimensional shape is formed by this cross section?

(1) rectangle
(2) triangle
(3) square
(4) circle

2 The endpoints of $\overline{AB}$ are $A(-5,3)$ and $B(7,-5)$. Point P is on $\overline{AB}$ such that $AP:PB = 3:1$. What are the coordinates of point P?

(1) $(-2,-3)$
(2) $(1,-1)$
(3) $(-2,1)$
(4) $(4,-3)$

Use this space for computations.

3 Zach placed the foot of an extension ladder 8 feet from the base of the house and extended the ladder 25 feet to reach the house. To the *nearest degree*, what is the measure of the angle the ladder makes with the ground?

(1) 18

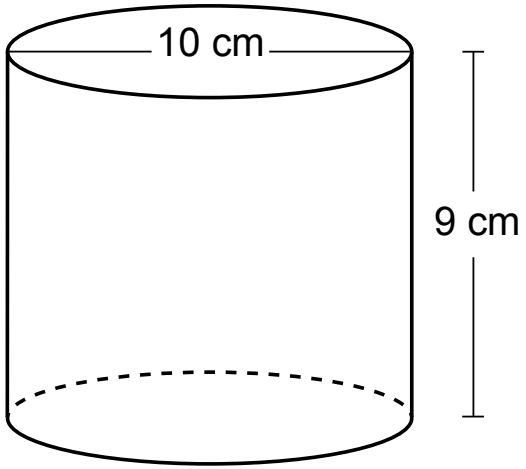
(2) 19

(3) 71

(4) 72

Use this space for computations.

4 Darnell models a cup with the cylinder below. He measured the diameter of the cup to be 10 cm and the height to be 9 cm.



If Darnell fills the cup with water to a height of 8 cm, what is the volume of the water in the cup, to the *nearest cubic centimeter*?

(1) 628
(2) 707
(3) 2513
(4) 2827

5 Which quadrilateral has diagonals that are always perpendicular?

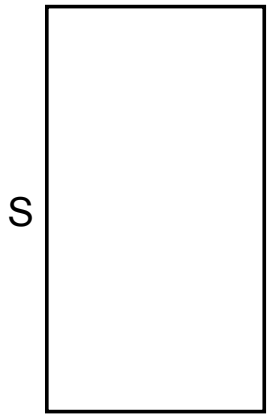
(1) rectangle
(2) rhombus
(3) trapezoid
(4) parallelogram

Use this space for computations.

6 Which regular polygon would carry onto itself after a rotation of 300° about its center?

(1) decagon
(2) nonagon
(3) octagon
(4) hexagon

7 The rectangle drawn below is continuously rotated about side *S*.



Which three-dimensional figure is formed by this rotation?

(1) rectangular prism
(2) square pyramid
(3) cylinder
(4) cone

Use this space for computations.

8 An equation of the line perpendicular to the line whose equation is $4x - 5y = 6$ and passes through the point $(-2,3)$ is

(1) $y + 3 = -\frac{5}{4}(x - 2)$

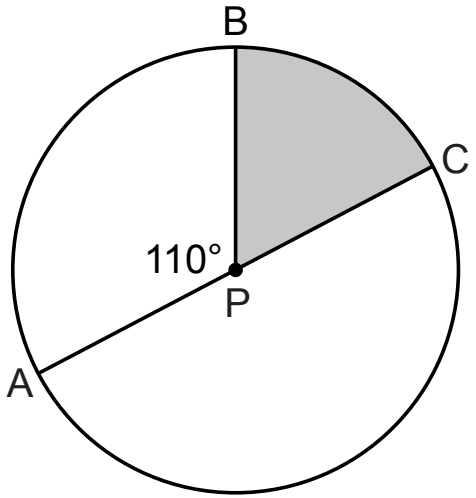
(2) $y - 3 = -\frac{5}{4}(x + 2)$

(3) $y + 3 = \frac{4}{5}(x - 2)$

(4) $y - 3 = \frac{4}{5}(x + 2)$

Use this space for computations.

9 In circle P below, diameter $\overline{AC}$ and radius $\overline{BP}$ are drawn such that $m\angle APB = 110°$.



If $AC = 12$, what is the area of shaded sector BPC?

(1) $\frac{7}{6}\pi$

(2) 7π

(3) 11π

(4) 28π

Use this space for computations.

10 In $\triangle ABC$, side $\overline{BC}$ is extended through C to D. If $m\angle A = 30°$ and $m\angle ACD = 110°$, what is the longest side of $\triangle ABC$?

(1) $\overline{AC}$

(2) $\overline{BC}$

(3) $\overline{AB}$

(4) $\overline{CD}$

11 Right triangle ACT has $m\angle A = 90°$. Which expression is always equivalent to $\cos T$?

(1) $\cos C$

(2) $\sin C$

(3) $\tan T$

(4) $\sin T$

12 A regular pyramid with a square base is made of solid glass. It has a base area of 36 cm^2 and a height of 10 cm. If the density of glass is 2.7 grams per cubic centimeter, the mass of the pyramid, in grams, is

(1) 120

(2) 324

(3) 360

(4) 972

Use this space for computations.

13 The equation of a circle is $x^2 + y^2 + 12x = -27$. What are the coordinates of the center and the length of the radius of the circle?

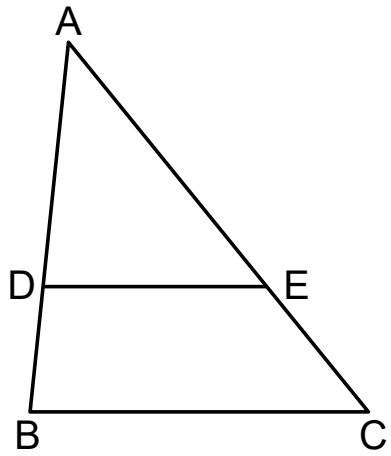
(1) center (6,0) and radius 3

(2) center (6,0) and radius 9

(3) center (−6,0) and radius 3

(4) center (−6,0) and radius 9

14 In triangle ABC below, D is a point on $\overline{AB}$ and E is a point on $\overline{AC}$, such that $\overline{DE} \parallel \overline{BC}$.



If $AD = 12$, $DB = 8$, and $EC = 10$, what is the length of $\overline{AC}$?

(1) 15

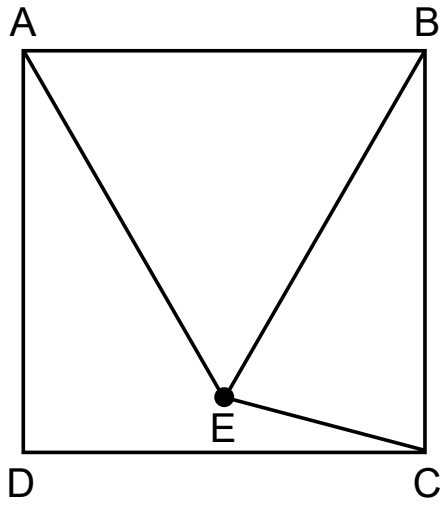
(2) 22

(3) 24

(4) 25

Use this space for computations.

15 In the diagram below, point E is located inside square $ABCD$ such that $\triangle ABE$ is equilateral, and $\overline{CE}$ is drawn.



What is $m\angle BEC$?

(1) 30°

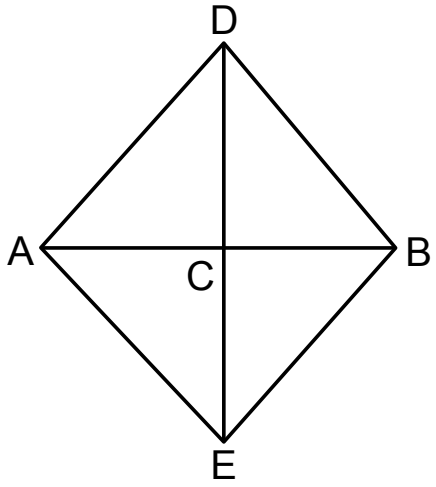
(2) 60°

(3) 75°

(4) 90°

Use this space for computations.

16 In the diagram below of quadrilateral $ADBE$, $\overline{DE}$ is the perpendicular bisector of $\overline{AB}$.



Which statement is always true?

(1) $\angle ADC \cong \angle BDC$

(2) $\angle EAC \cong \angle DAC$

(3) $\overline{AD} \cong \overline{BE}$

(4) $\overline{AE} \cong \overline{AD}$

17 What is the image of (4,3) after a reflection over the line $y = 1$?

(1) $(-2,3)$

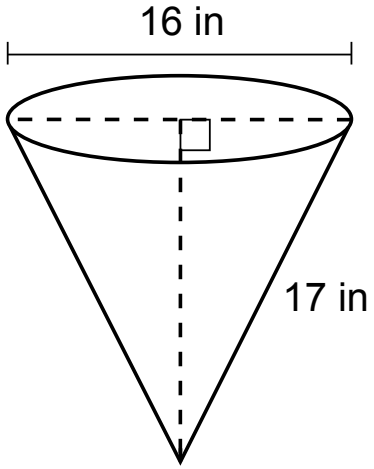
(2) $(-4,3)$

(3) $(4,-1)$

(4) $(4,-3)$

Use this space for computations.

18 In the diagram below, a cone has a diameter of 16 inches and a slant height of 17 inches.



What is the volume of the cone, in cubic inches?

(1) 320π

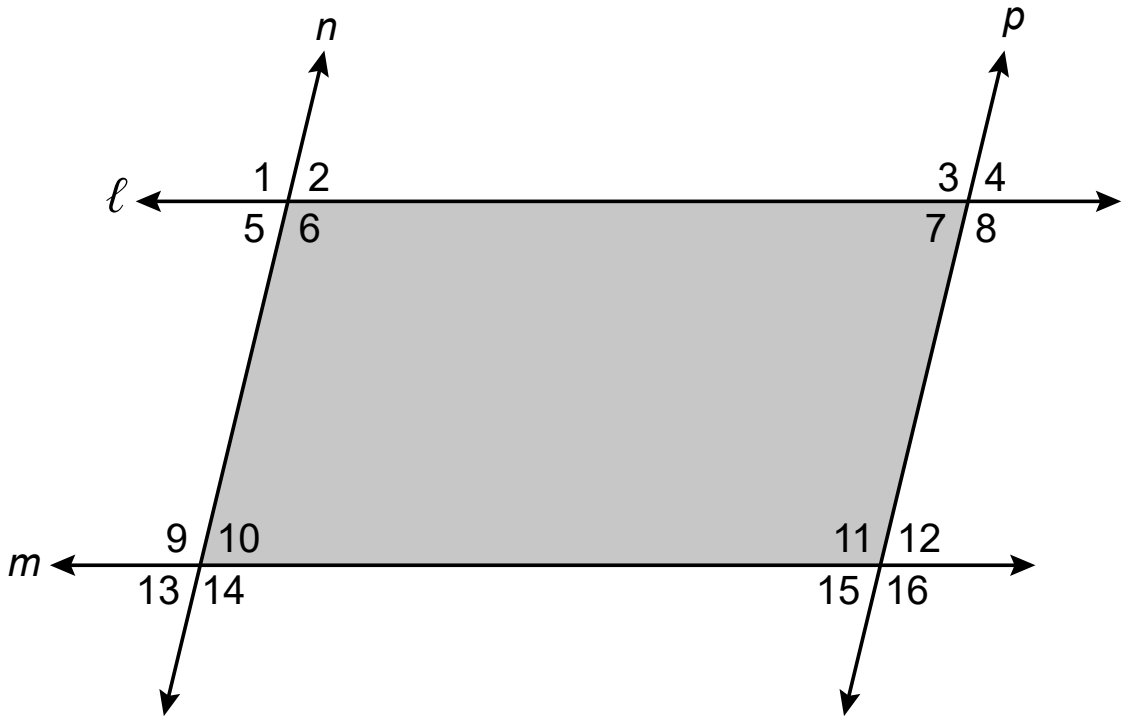
(2) 363π

(3) 960π

(4) 1280π

Use this space for computations.

19 In the diagram below, lines ℓ and m intersect lines n and p to create the shaded quadrilateral as shown.



Which congruence statement would be sufficient to prove the quadrilateral is a parallelogram?

(1) $\angle 1 \cong \angle 6$ and $\angle 9 \cong \angle 14$

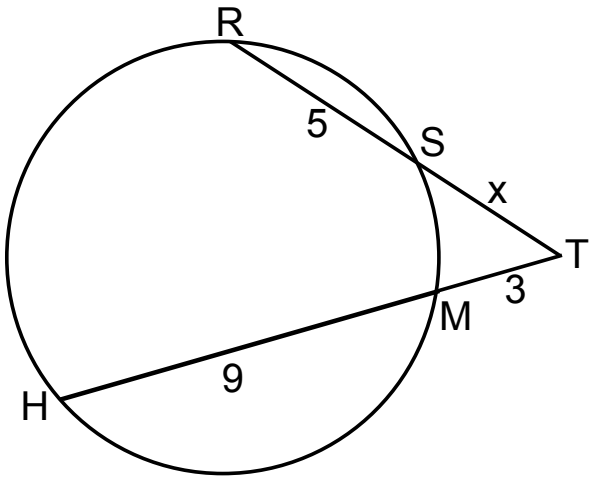
(2) $\angle 5 \cong \angle 10$ and $\angle 6 \cong \angle 9$

(3) $\angle 5 \cong \angle 7$ and $\angle 10 \cong \angle 15$

(4) $\angle 6 \cong \angle 9$ and $\angle 9 \cong \angle 11$

Use this space for computations.

20 In the circle below, secants $\overline{TSR}$ and $\overline{TMH}$ intersect at T, $SR = 5$, $HM = 9$, $TM = 3$, and $TS = x$.



Which equation could be used to find the value of x?

(1) $x(x + 5) = 36$

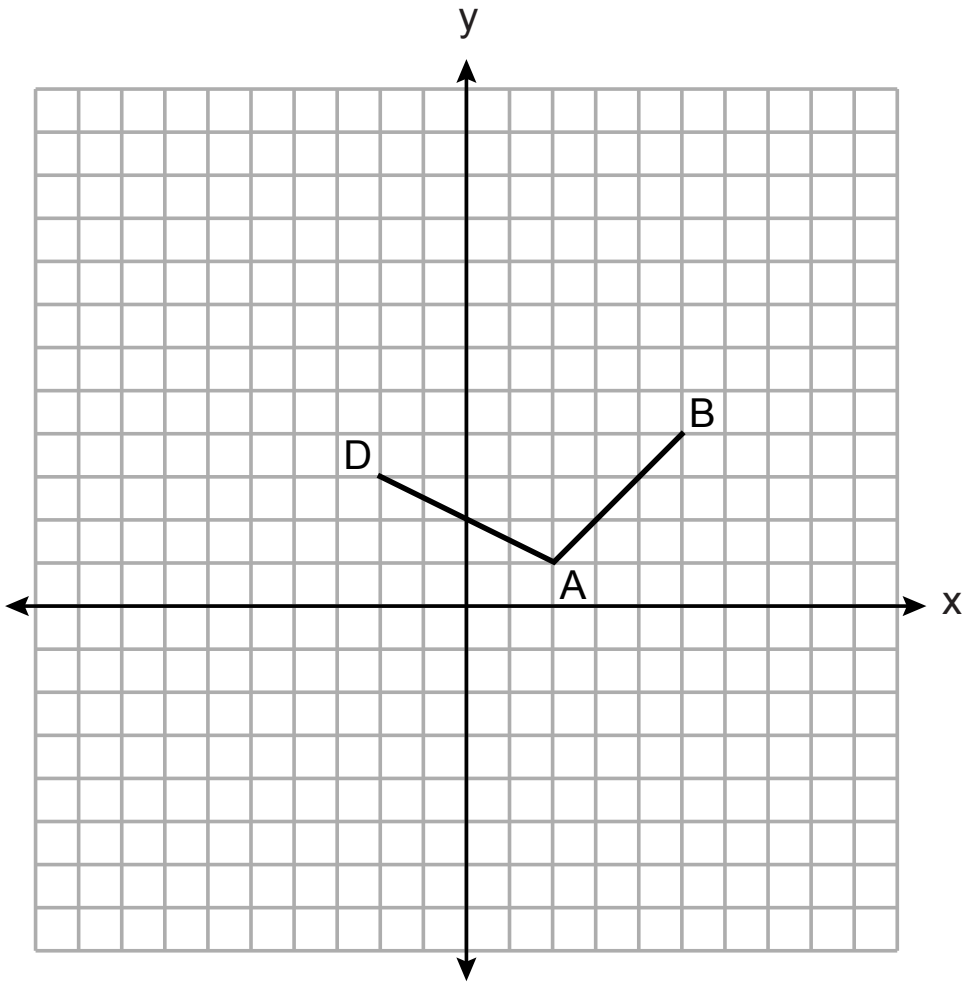
(2) $x(x + 5) = 27$

(3) $3x = 45$

(4) $5x = 27$

Use this space for computations.

21 On the set of axes below, the coordinates of three vertices of trapezoid $ABCD$ are $A(2,1)$, $B(5,4)$, and $D(-2,3)$.



Which point could be vertex C?

(1) (1,5)

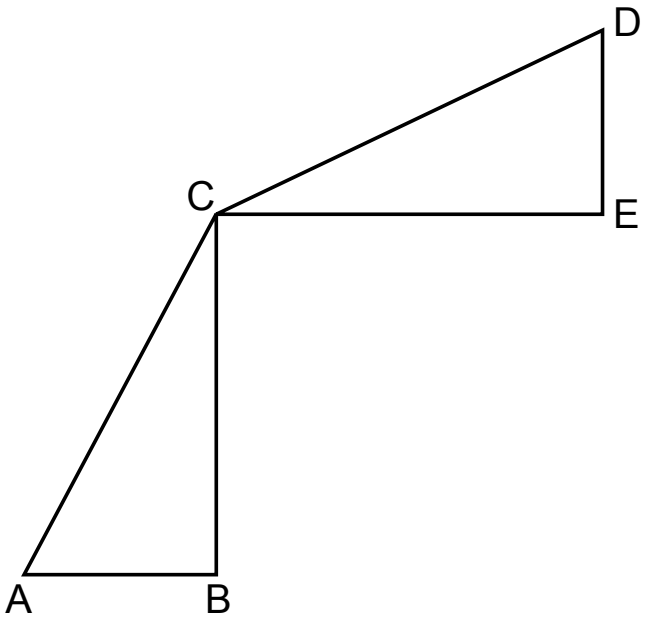
(2) (4,10)

(3) (−1,6)

(4) (−3,8)

Use this space for computations.

22 In the diagram below, $\triangle ABC \cong \triangle DEC$.



Which transformation will map $\triangle ABC$ onto $\triangle DEC$?

(1) a rotation

(2) a line reflection

(3) a translation followed by a dilation

(4) a line reflection followed by a second line reflection

Use this space for computations.

23 If $\triangle TAP$ is dilated by a scale factor of 0.5, which statement about the image, $\triangle T'A'P'$, is true?

(1) $m\angle T'A'P' = \frac{1}{2}(m\angle TAP)$

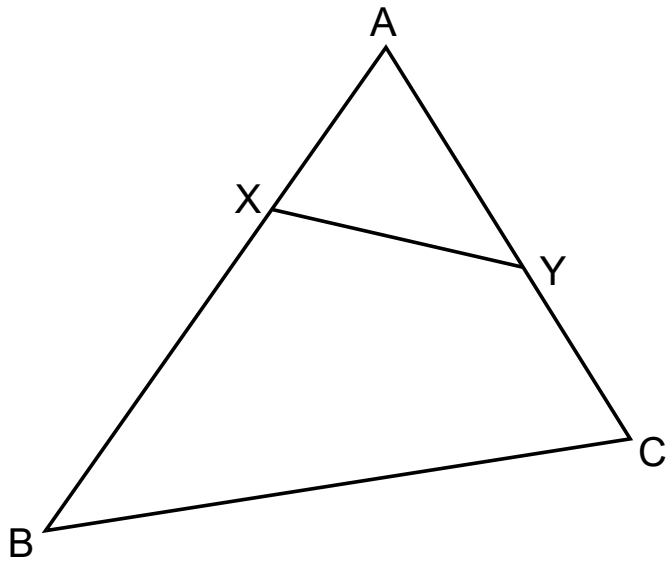
(2) $m\angle T'A'P' = 2(m\angle TAP)$

(3) $TA = 2(T'A')$

(4) $TA = \frac{1}{2}(T'A')$

Use this space for computations.

24 In the diagram below of $\triangle ABC$, X and Y are points on $\overline{AB}$ and $\overline{AC}$, respectively, such that $m\angle AYX = m\angle B$.



Which statement is *not* always true?

(1) $\frac{AX}{AC} = \frac{XY}{CB}$

(2) $\frac{AY}{AB} = \frac{AX}{AC}$

(3) $(AY)(CB) = (XY)(AB)$

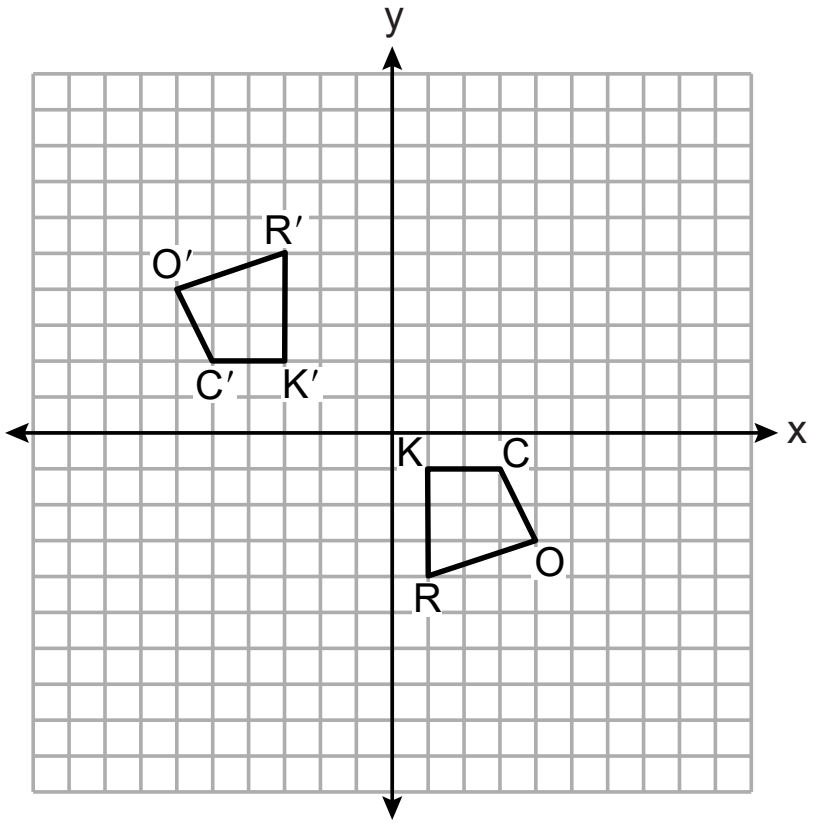
(4) $(AY)(AB) = (AC)(AX)$

GO RIGHT ON TO THE NEXT PAGE ⇨

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 On the set of axes below, congruent quadrilaterals $ROCK$ and $R'O'C'K'$ are graphed.



Question 25 is continued on the next page.

Question 25 continued

Describe a sequence of transformations that would map quadrilateral $ROCK$ onto quadrilateral $R'O'C'K'$.

26 In triangle CEM, $CE = 3x + 10$, $ME = 5x - 14$, and $CM = 2x - 6$.

Determine and state the value of x that would make $\triangle CEM$ an isosceles triangle with the vertex angle at E.

Work space for question 26 is continued on the next page.

Question 26 continued

27 A flagpole casts a shadow on the ground 91 feet long, with a 53° angle of elevation from the end of the shadow to the top of the flagpole.

Determine and state, to the *nearest tenth of a foot*, the height of the flagpole.

Work space for question 27 is continued on the next page.

Question 27 continued

28 A man is spray-painting the tops of 10 patio tables. Five tables have round tops, with diameters of 4 feet, and five tables have rectangular tops, with dimensions of 4 feet by 6 feet. A can of spray paint covers 25 square feet. How many cans of spray paint must be purchased to paint all of the tabletops?

Work space for question 28 is continued on the next page.

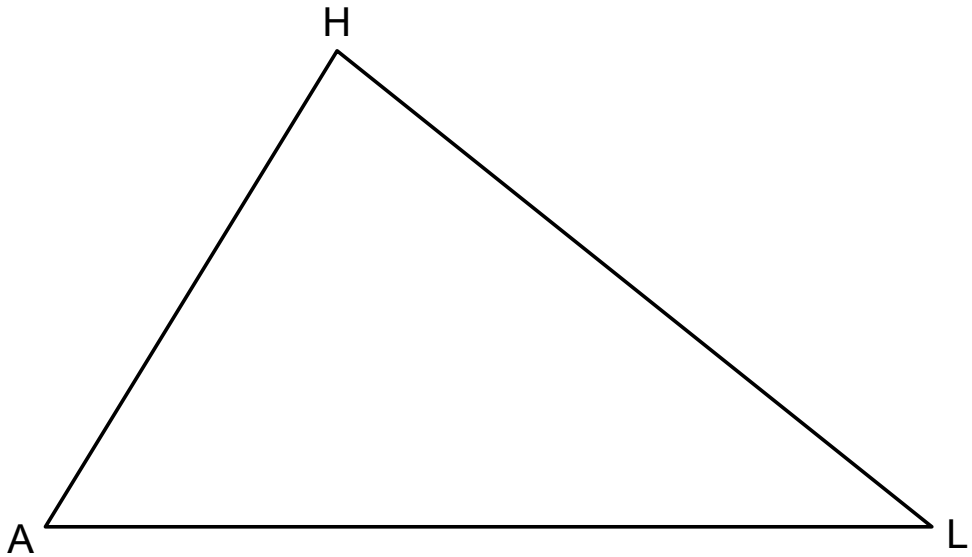
Question 28 continued

29 Using a compass and straightedge, construct a midsegment of $\triangle AHL$ on the next page.

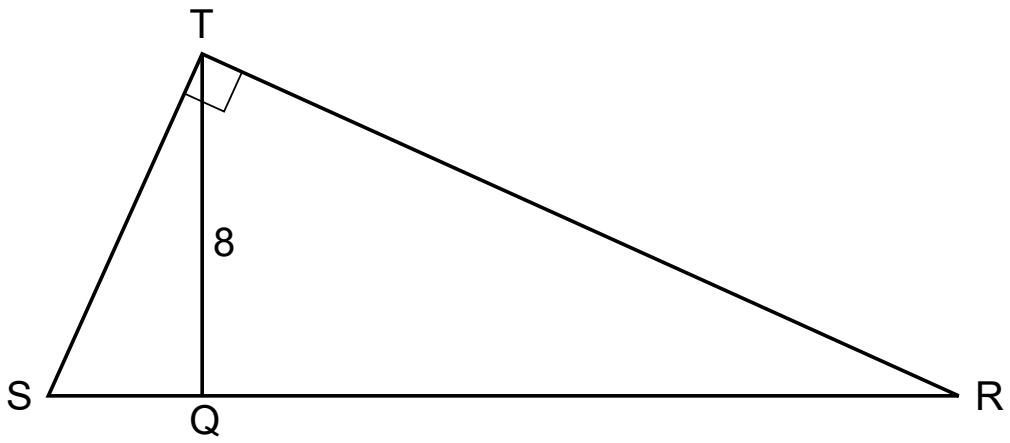
[Leave all construction marks.]

Question 29 is continued on the next page.

Question 29 continued



30 Right triangle STR is shown below, with $m\angle T = 90°$. Altitude $\overline{TQ}$ is drawn to $\overline{SQR}$, and $TQ = 8$.

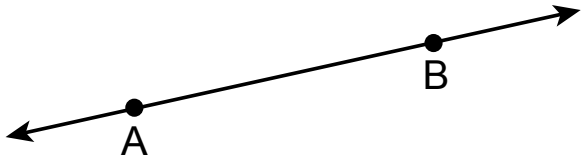


If the ratio SQ:QR is 1:4, determine and state the length of $\overline{SR}$.

Work space for question 30 is continued on the next page.

Question 30 continued

31 Line AB is dilated by a scale factor of 2 centered at point A.



Evan thinks that the dilation of $\overleftrightarrow{AB}$ will result in a line parallel to $\overleftrightarrow{AB}$, not passing through points A or B.

Nathan thinks that the dilation of $\overleftrightarrow{AB}$ will result in the same line, $\overleftrightarrow{AB}$.

Who is correct?

Explain why.

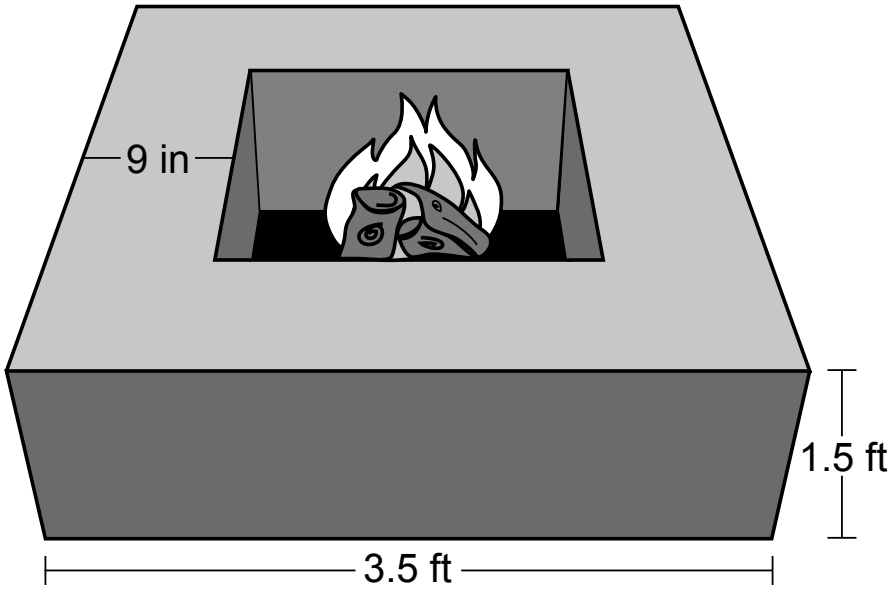
Work space for question 31 is continued on the next page.

Question 31 continued

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Josh is making a square-based fire pit out of concrete for his backyard, as modeled by the right prism below. He plans to make the outside walls of the fire pit 3.5 feet on each side with a height of 1.5 feet. The concrete walls of the fire pit are going to be 9 inches thick.

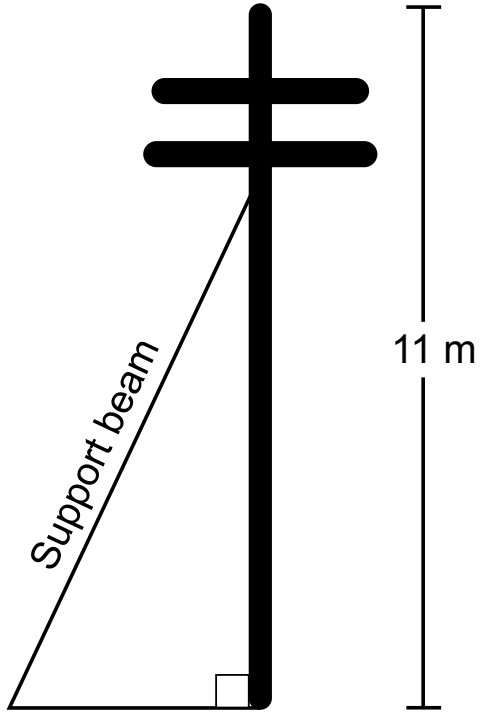


Question 32 is continued on the next page.

Question 32 continued

If a bag of concrete mix will fill 0.6 ft^3, determine and state the minimum number of bags needed to build the fire pit.

33 A telephone pole 11 meters tall needs to be stabilized with a support beam, as modeled below.



Two conditions for proper support are:

- The beam reaches the telephone pole at 70% of the telephone pole's height above the ground.
- The beam forms a 65° angle with the ground.

Question 33 is continued on the next page.

Determine and state, to the *nearest tenth of a meter*, the length of the support beam that meets these conditions for this telephone pole.

Determine and state, to the *nearest tenth of a meter*, how far the support beam must be placed from the base of the pole to meet the conditions.

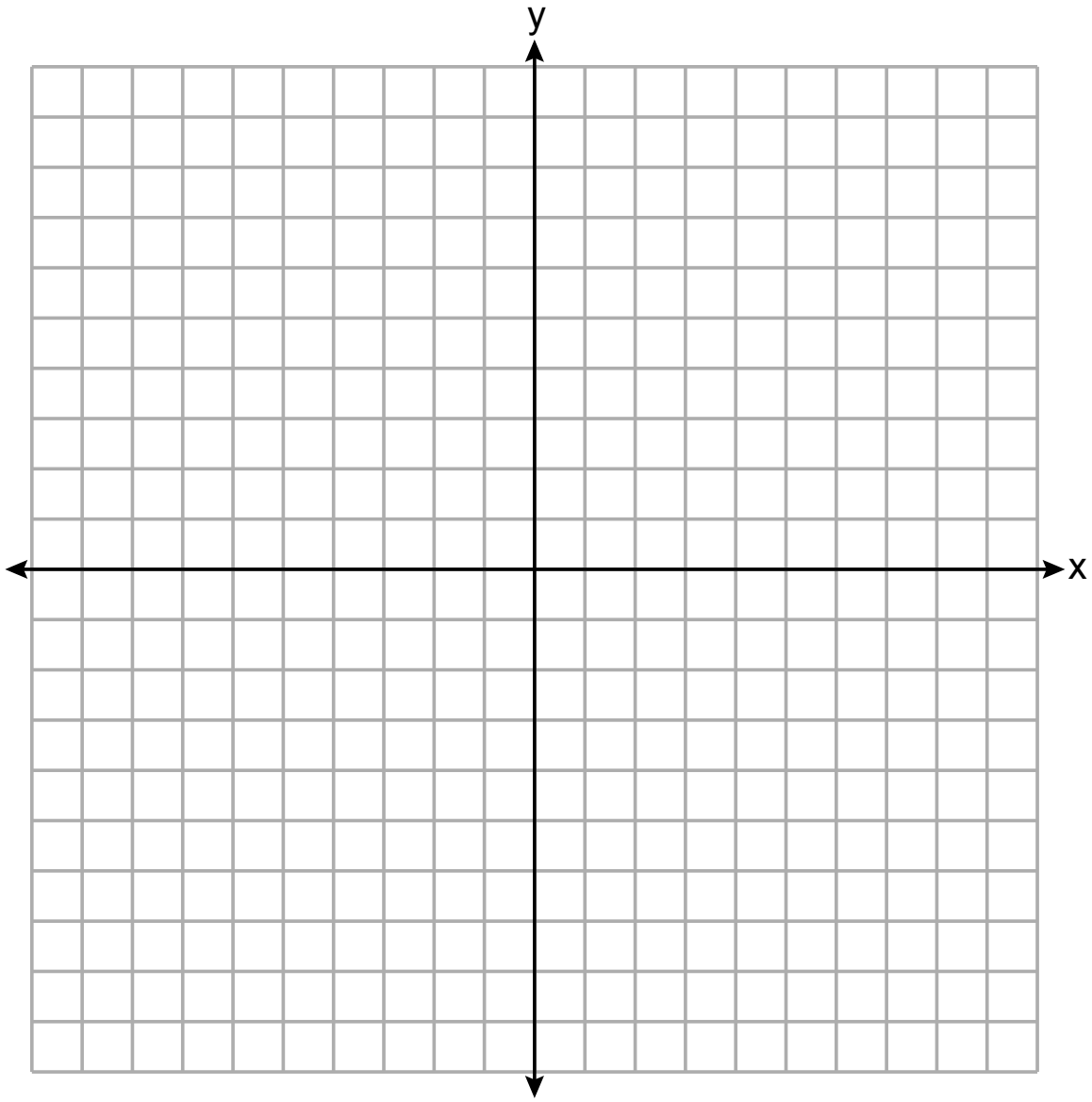
34 The coordinates of the vertices of quadrilateral $ABCD$ are $A(0,4)$, $B(3,8)$, $C(8,3)$, and $D(5,-1)$.

Prove that $ABCD$ is a parallelogram, but *not* a rectangle.

[The use of the set of axes on the next page is optional.]

The set of axes for question 34 is on the next page.

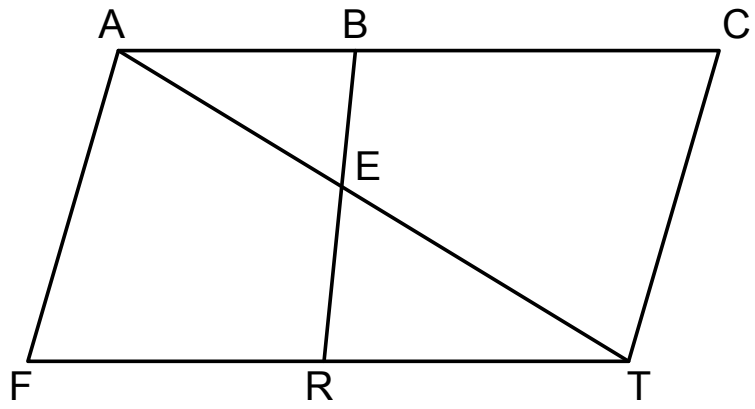
Question 34 continued



Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 In the diagram below of quadrilateral $FACT$, $\overline{BR}$ intersects diagonal $\overline{AT}$ at E, $\overline{AF} \parallel \overline{CT}$, and $\overline{AF} \cong \overline{CT}$.



Prove: $(AB)(TE) = (AE)(TR)$

Work space for question 35 is continued on the next page.

Question 35 continued

Work space for question 35 is continued on the next page.

Question 35 continued

Scrap Graph Paper — this sheet will *not* be scored.

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High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - \mathrm{r}}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$