

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

**Friday, June 21, 2024** — 9:15 a.m. to 12:15 p.m., only

**Student Name:** \_\_\_\_\_

**School Name:** \_\_\_\_\_

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.



This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice ...**

**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

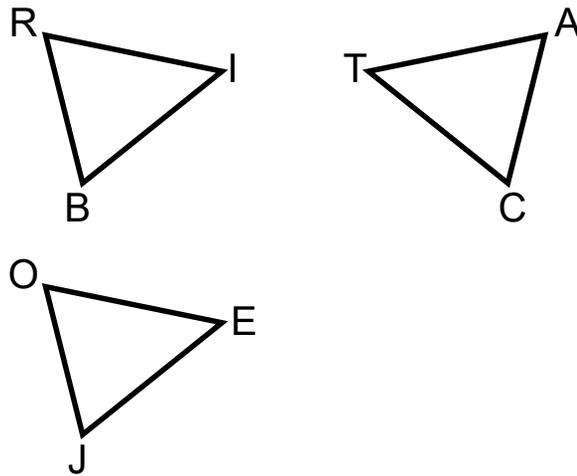
**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for  
computations.

- 1 In the diagram below,  $\triangle BRI$  is the image of  $\triangle JOE$  after a translation.  
Triangle  $CAT$  is the image of  $\triangle BRI$  after a line reflection.



Which statement is always true?

- (1)  $\angle R \cong \angle T$                       (3)  $\overline{JE} \cong \overline{RI}$   
(2)  $\angle J \cong \angle A$                       (4)  $\overline{OE} \cong \overline{AT}$

**Use this space for  
computations.**

**2** A right cylinder is cut parallel to its base. The shape of this cross section is a

(1) cone

(3) triangle

(2) circle

(4) rectangle

**3** What is the minimum number of degrees that a regular hexagon must rotate about its center to carry it onto itself?

(1)  $45^\circ$

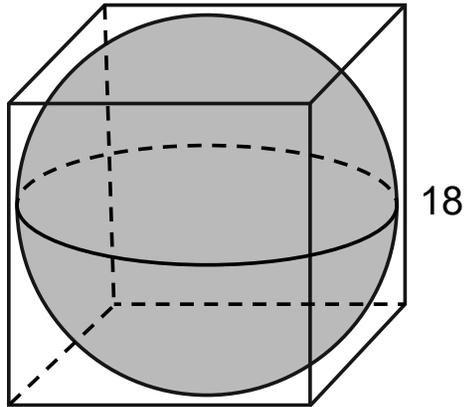
(3)  $60^\circ$

(2)  $72^\circ$

(4)  $120^\circ$

**Use this space for  
computations.**

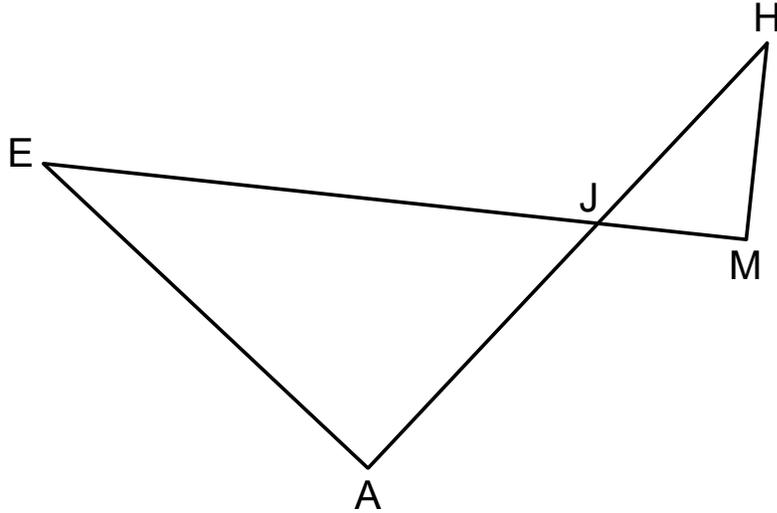
- 4 In the diagram below, a sphere is inscribed inside a cube. The cube has edge lengths of 18.



What is the volume of the sphere, in terms of  $\pi$ ?

- (1)  $108\pi$                       (3)  $972\pi$   
(2)  $432\pi$                       (4)  $7776\pi$

- 5 In the diagram below,  $\overline{EM}$  intersects  $\overline{HA}$  at  $J$ ,  $\overline{EA} \perp \overline{HA}$ , and  $\overline{EM} \perp \overline{HM}$ .



Use this space for  
computations.

If  $EA = 7.2$ ,  $EJ = 9$ ,  $AJ = 5.4$ , and  $HM = 3.29$ , what is the length of  $\overline{MJ}$ , to the *nearest hundredth*?

- |          |          |
|----------|----------|
| (1) 2.47 | (3) 4.11 |
| (2) 2.63 | (4) 4.39 |

**Use this space for  
computations.**

**6** Which equation represents the line that passes through the point  $(2, -7)$  and is perpendicular to the line whose equation is  $y = \frac{3}{4}x + 4$ ?

(1)  $y + 7 = \frac{3}{4}(x - 2)$

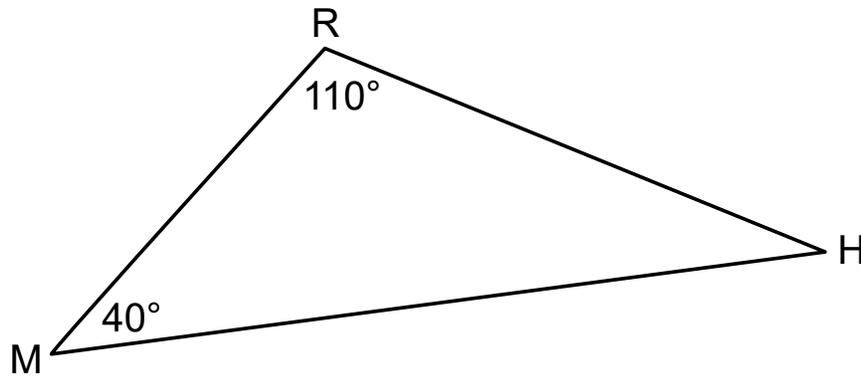
(3)  $y + 7 = -\frac{4}{3}(x - 2)$

(2)  $y - 7 = \frac{3}{4}(x + 2)$

(4)  $y - 7 = -\frac{4}{3}(x + 2)$

7 In  $\triangle RHM$  below,  $m\angle R = 110^\circ$  and  $m\angle M = 40^\circ$ .

Use this space for  
computations.

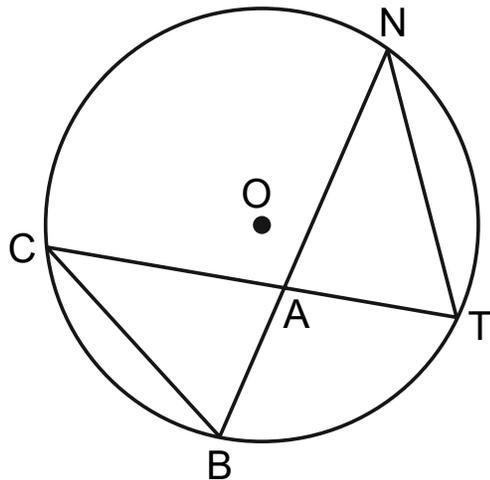


If  $\triangle RHM$  is reflected over side  $\overline{HM}$  to form quadrilateral  $RHR'M$ , which statement is always true?

- (1) Quadrilateral  $RHR'M$  is a parallelogram.
- (2)  $m\angle MHR' = 40^\circ$
- (3)  $m\angle HMR' = 40^\circ$
- (4)  $\overline{MR} \cong \overline{HR'}$



- 9 In circle  $O$  below, chords  $\overline{CT}$  and  $\overline{BN}$  intersect at point  $A$ . Chords  $\overline{CB}$  and  $\overline{NT}$  are drawn.



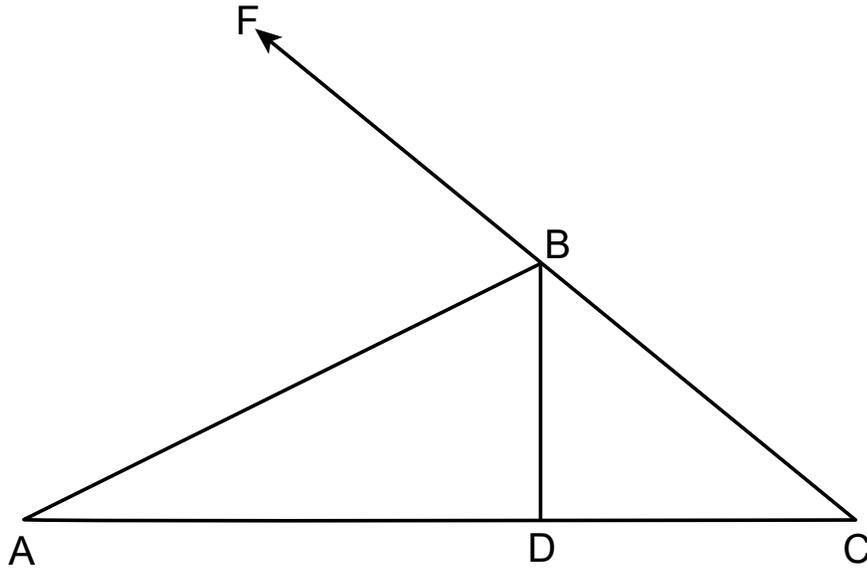
Use this space for  
computations.

Which statement is always true?

- (1)  $\frac{NT}{TA} = \frac{CB}{BA}$                       (3)  $\frac{NA}{AB} = \frac{TA}{AC}$   
(2)  $\angle BAC \cong \angle ATN$                       (4)  $\angle BCA \cong \angle NTA$

10 In the diagram below of  $\triangle ABC$ ,  $\overrightarrow{CBF}$  is drawn,  $\overline{AB}$  bisects  $\angle FBD$ , and  $\overline{BD} \perp \overline{AC}$ .

Use this space for computations.

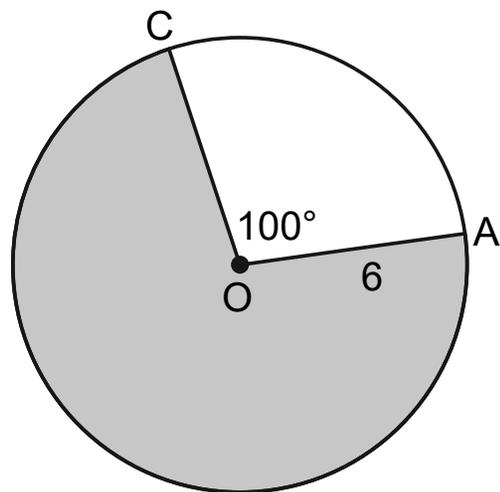


If  $m\angle C = 42^\circ$ , what is  $m\angle A$ ?

- |                |                |
|----------------|----------------|
| (1) $24^\circ$ | (3) $48^\circ$ |
| (2) $33^\circ$ | (4) $66^\circ$ |

11 In circle  $O$  below,  $OA = 6$ , and  $m\angle COA = 100^\circ$ .

Use this space for  
computations.



What is the area of the shaded sector?

- (1)  $10\pi$                       (3)  $\frac{10\pi}{3}$   
(2)  $26\pi$                       (4)  $\frac{26\pi}{3}$



**Use this space for  
computations.**

**14** Triangle  $KLM$  is dilated by a scale factor of 3 to map onto triangle  $DRS$ . Which statement is *not* always true?

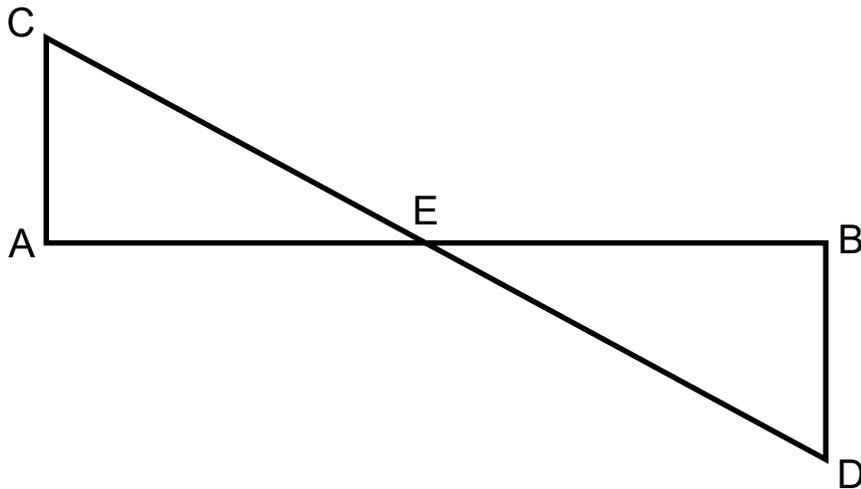
- (1)  $\angle K \cong \angle D$
- (2)  $KM = \frac{1}{3}DS$
- (3) The area of  $\triangle DRS$  is 3 times the area of  $\triangle KLM$ .
- (4) The perimeter of  $\triangle DRS$  is 3 times the perimeter of  $\triangle KLM$ .

**15** A rectangle with dimensions of 4 feet by 7 feet is continuously rotated about one of its 4-foot sides. The resulting three-dimensional object is a

- (1) cylinder with a height of 7 feet and a base radius of 4 feet.
- (2) cylinder with a height of 4 feet and a base radius of 7 feet.
- (3) cone with a height of 7 feet and a base radius of 7 feet.
- (4) cone with a height of 4 feet and a base radius of 7 feet.



- 19 In the diagram below,  $\overline{AB}$  and  $\overline{CD}$  intersect at  $E$ , and  $\overline{CA}$  and  $\overline{DB}$  are drawn.



Use this space for  
computations.

If  $\overline{CA} \parallel \overline{BD}$ , which statement is always true?

- (1)  $\overline{AE} \cong \overline{BE}$                       (3)  $\triangle AEC \sim \triangle BED$   
(2)  $\overline{CA} \cong \overline{DB}$                       (4)  $\triangle AEC \cong \triangle BED$

**Use this space for  
computations.**

**20** If  $\sin(3x + 9)^\circ = \cos(5x - 7)^\circ$ , what is the value of  $x$ ?

(1) 8

(3) 33

(2) 11

(4) 42

**21** Which set of integers could represent the lengths of the sides of an isosceles triangle?

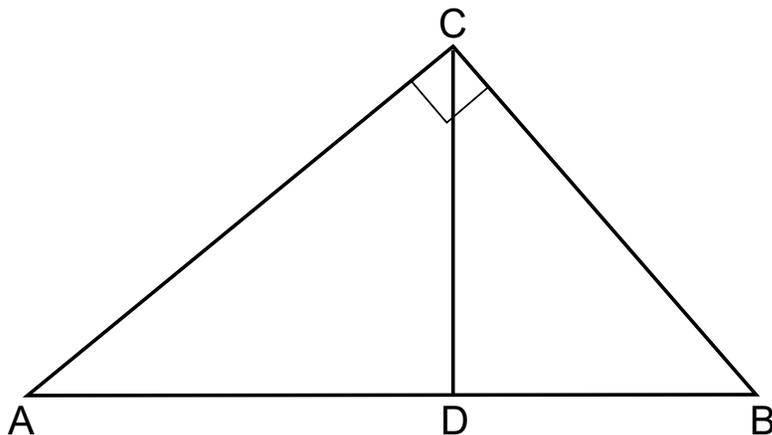
(1) {1, 1, 3}

(3) {3, 3, 6}

(2) {2, 2, 5}

(4) {4, 4, 7}

- 22 In the diagram shown below, altitude  $\overline{CD}$  is drawn to the hypotenuse of right triangle  $ABC$ .



Use this space for  
computations.

Which equation can always be used to find the length of  $\overline{AC}$ ?

(1)  $\frac{AC}{CD} = \frac{CD}{AD}$

(3)  $\frac{AC}{CD} = \frac{CD}{BC}$

(2)  $\frac{CD}{AC} = \frac{AC}{AB}$

(4)  $\frac{AB}{AC} = \frac{AC}{AD}$

**Use this space for  
computations.**

**23** Which congruence statement is sufficient to prove parallelogram *MARK* is a rhombus?

(1)  $\overline{MA} \cong \overline{MK}$

(3)  $\angle K \cong \angle A$

(2)  $\overline{MA} \cong \overline{KR}$

(4)  $\angle R \cong \angle A$

**24** A line whose equation is  $y = -2x + 3$  is dilated by a scale factor of 4 centered at  $(0,3)$ . Which equation represents the image of the line after the dilation?

(1)  $y = -2x + 3$

(3)  $y = -8x + 3$

(2)  $y = -2x + 12$

(4)  $y = -8x + 12$

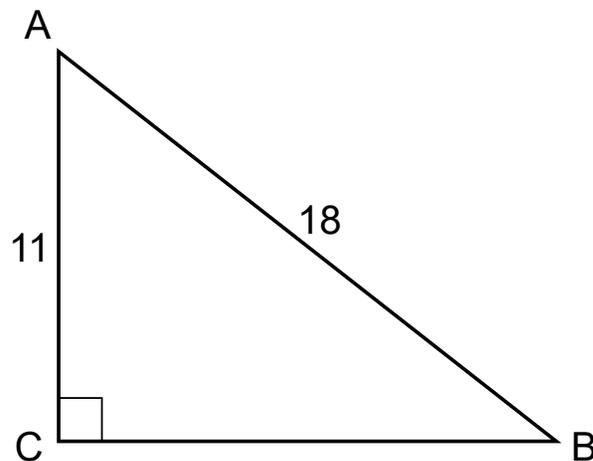
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## Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 In  $\triangle ABC$  below,  $m\angle C = 90^\circ$ ,  $AC = 11$ , and  $AB = 18$ .



Determine and state the measure of angle A, to the *nearest degree*.

Work space for question 25 is continued on the next page.

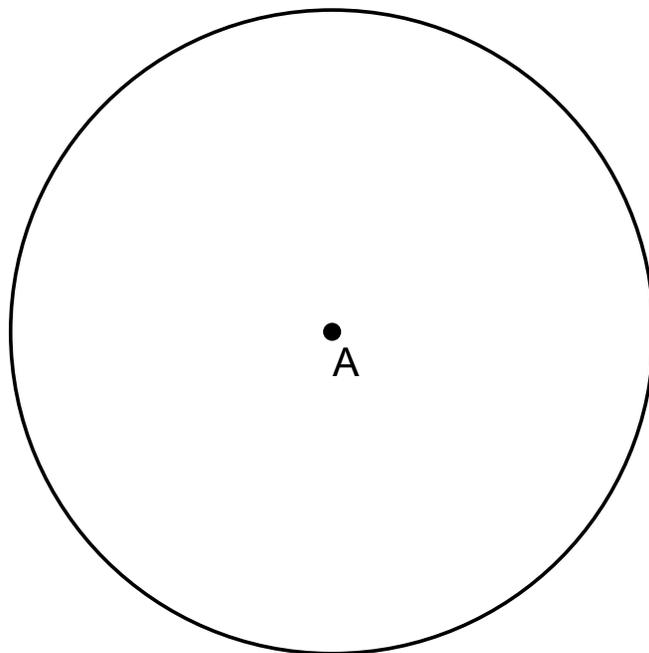
**Question 25 continued**

**26** Use a compass and straightedge to construct an equilateral triangle inscribed in circle  $A$  on the next page.

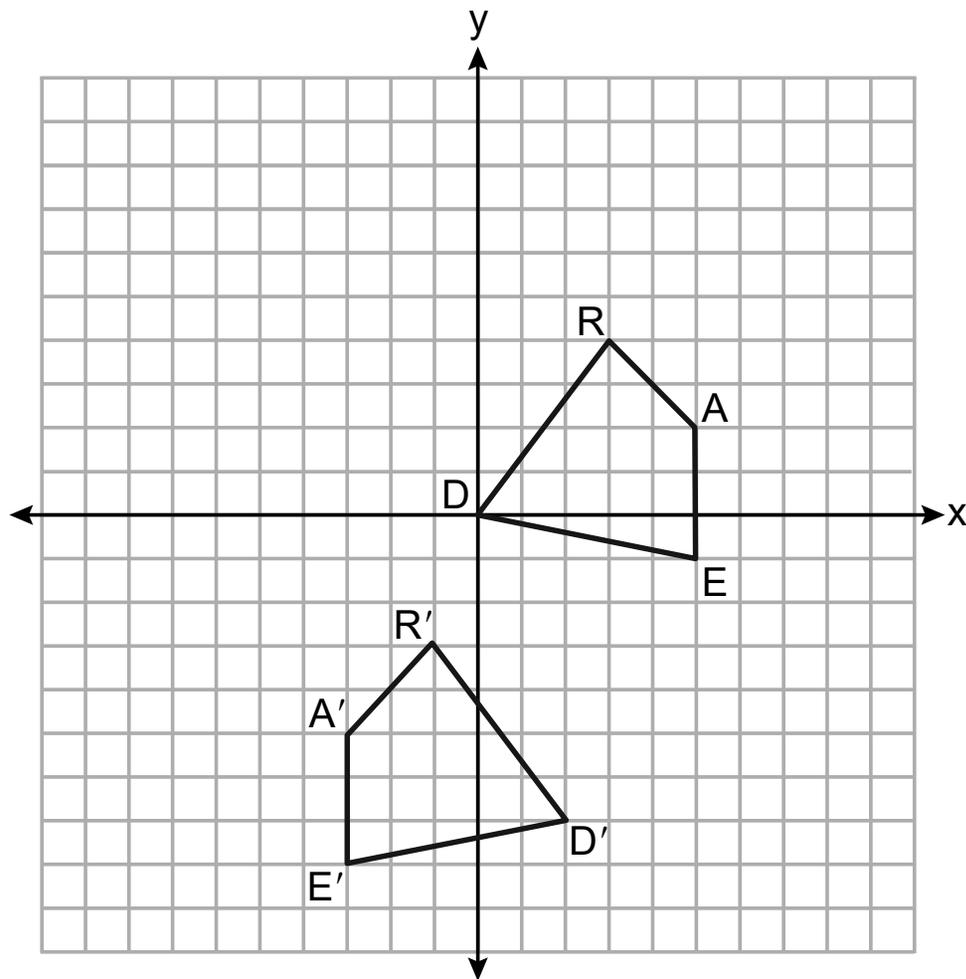
[Leave all construction marks.]

**Question 26 is continued on the next page.**

**Question 26 continued**



27 Quadrilateral  $DEAR$  and its image, quadrilateral  $D'E'A'R'$ , are graphed on the set of axes below.

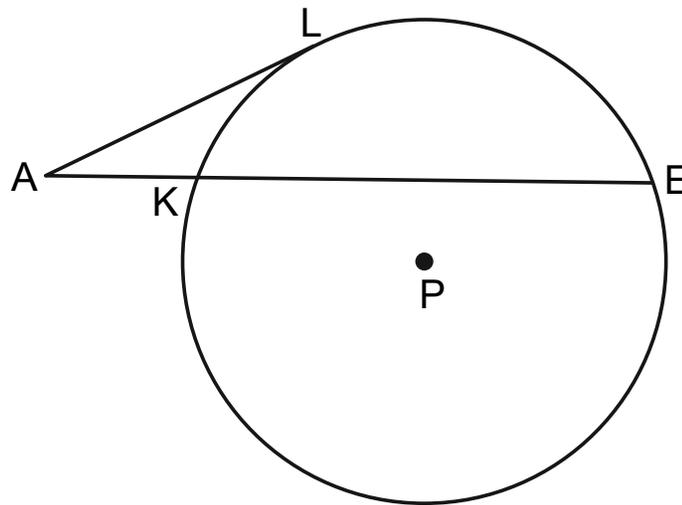


Describe a sequence of transformations that maps quadrilateral  $DEAR$  onto quadrilateral  $D'E'A'R'$ .

Work space for question 27 is continued on the next page.

**Question 27 continued**

28 In circle  $P$  below, tangent  $\overline{AL}$  and secant  $\overline{AKE}$  are drawn.



If  $AK = 12$  and  $KE = 36$ , determine and state the length of  $\overline{AL}$ .

Work space for question 28 is continued on the next page.

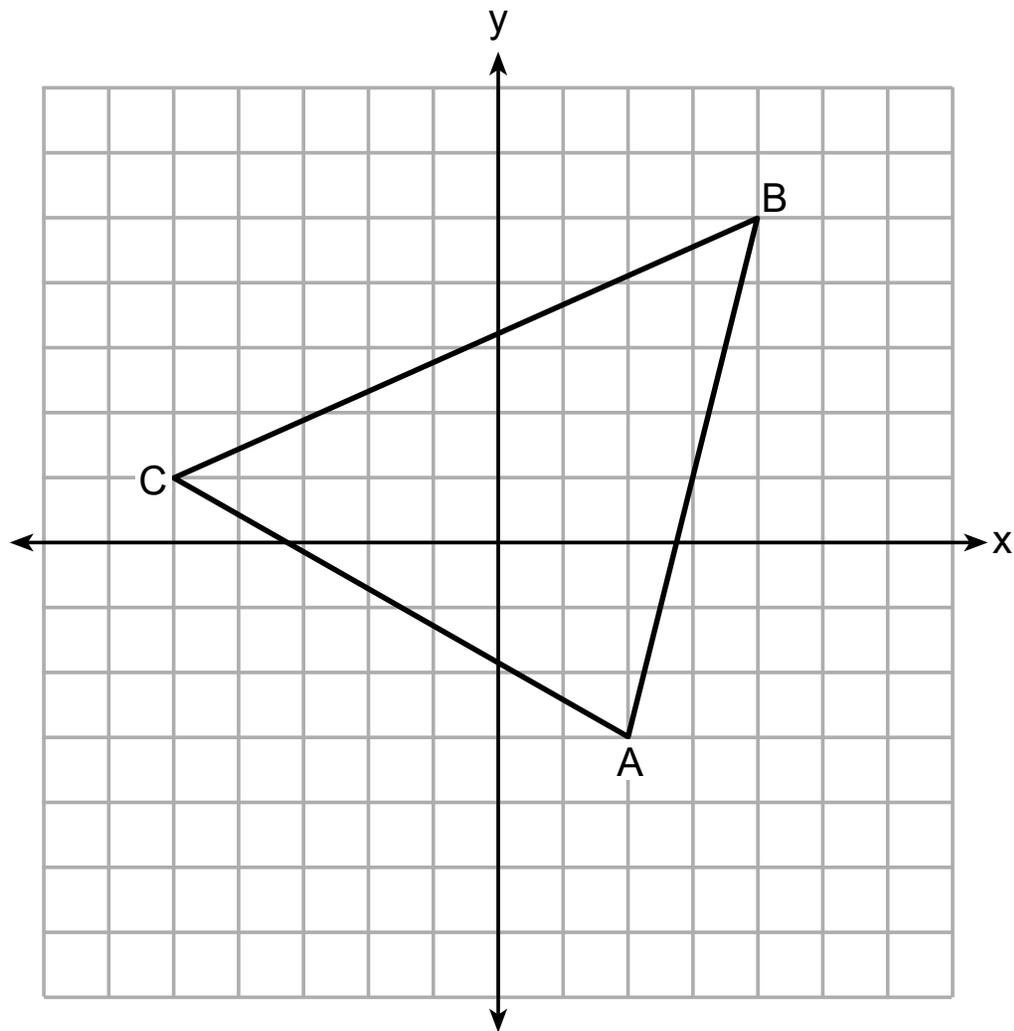
**Question 28 continued**

**29** The equation of a circle is  $x^2 + y^2 + 8x - 6y + 7 = 0$ . Determine and state the coordinates of the center and the length of the radius of the circle.

**Work space for question 29 is continued on the next page.**

**Question 29 continued**

**30** On the set of axes below,  $\triangle ABC$  is drawn with vertices that have coordinates  $A(2,-3)$ ,  $B(4,5)$ , and  $C(-5,1)$ .

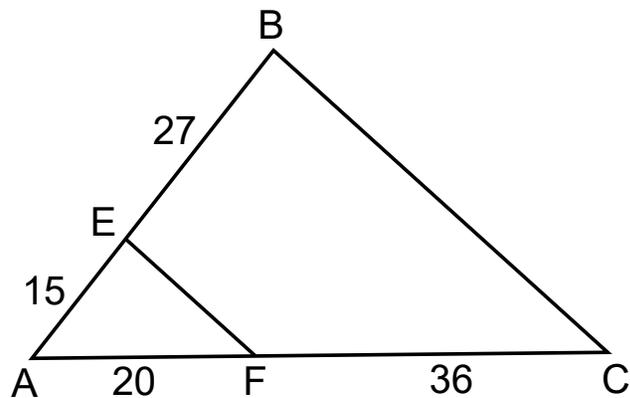


Determine and state the area of  $\triangle ABC$ .

**Work space for question 30 is continued on the next page.**

**Question 30 continued**

31 In the diagram below,  $AE = 15$ ,  $EB = 27$ ,  $AF = 20$ , and  $FC = 36$ .



Explain why  $\overline{EF} \parallel \overline{BC}$ .

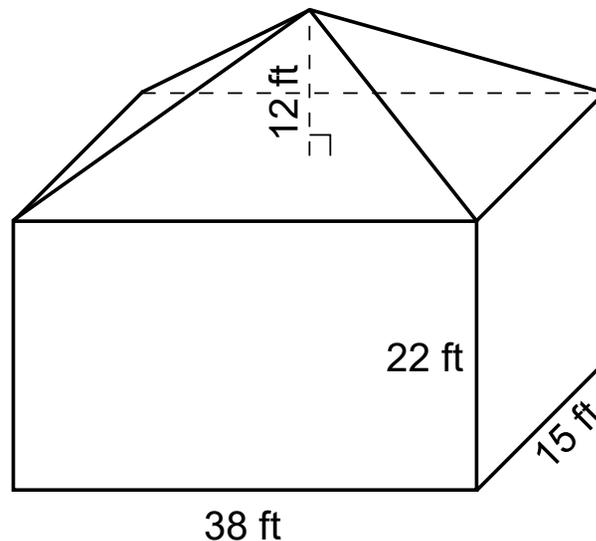
Work space for question 31 is continued on the next page.

**Question 31 continued**

### Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 32** A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.

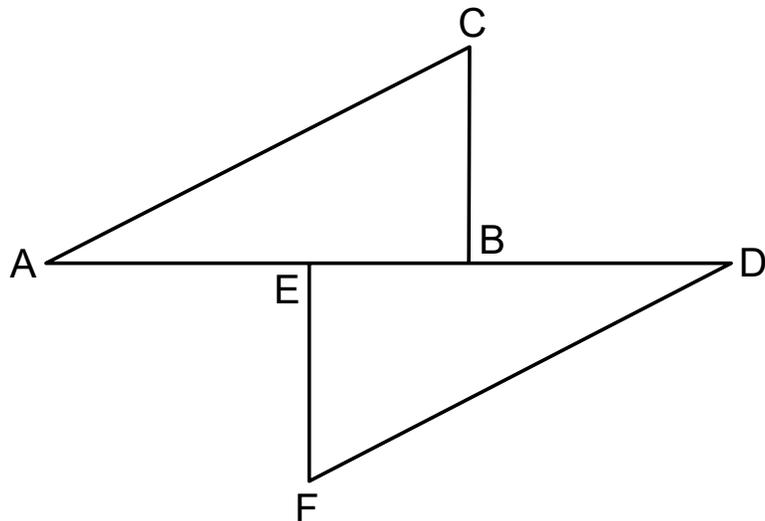


Question 32 is continued on the next page.

### Question 32 continued

An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

33 Given:  $\triangle ABC$ ,  $\triangle DEF$ ,  $\overline{AB} \perp \overline{BC}$ ,  $\overline{DE} \perp \overline{EF}$ ,  $\overline{AE} \cong \overline{DB}$ , and  $\overline{AC} \parallel \overline{FD}$

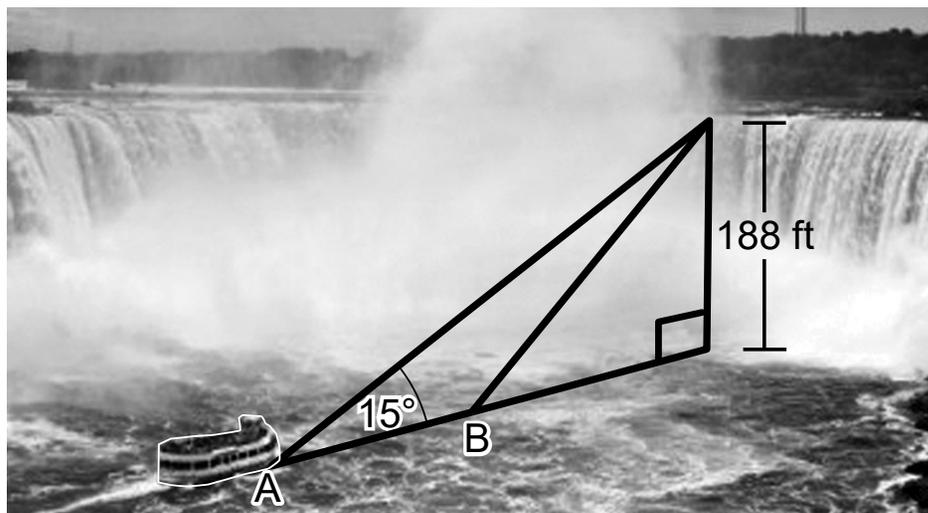


Prove:  $\triangle ABC \cong \triangle DEF$

Work space for question 33 is continued on the next page.

**Question 33 continued**

- 34 In the diagram below, a boat at point  $A$  is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point  $A$  to the top of the waterfall is  $15^\circ$ .



After the boat travels toward the falls, the angle of elevation at point  $B$  to the top of the waterfall is  $23^\circ$ . Determine and state, to the *nearest foot*, the distance the boat traveled from point  $A$  to point  $B$ .

Work space for question 34 is continued on the next page.

**Question 34 continued**

## Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

**35** Triangle  $JOE$  has vertices whose coordinates are  $J(4,6)$ ,  $O(-2,4)$ , and  $E(6,0)$ .

Prove that  $\triangle JOE$  is isosceles.

[The use of the set of axes on page 44 is optional.]

Question 35 is continued on the next page.

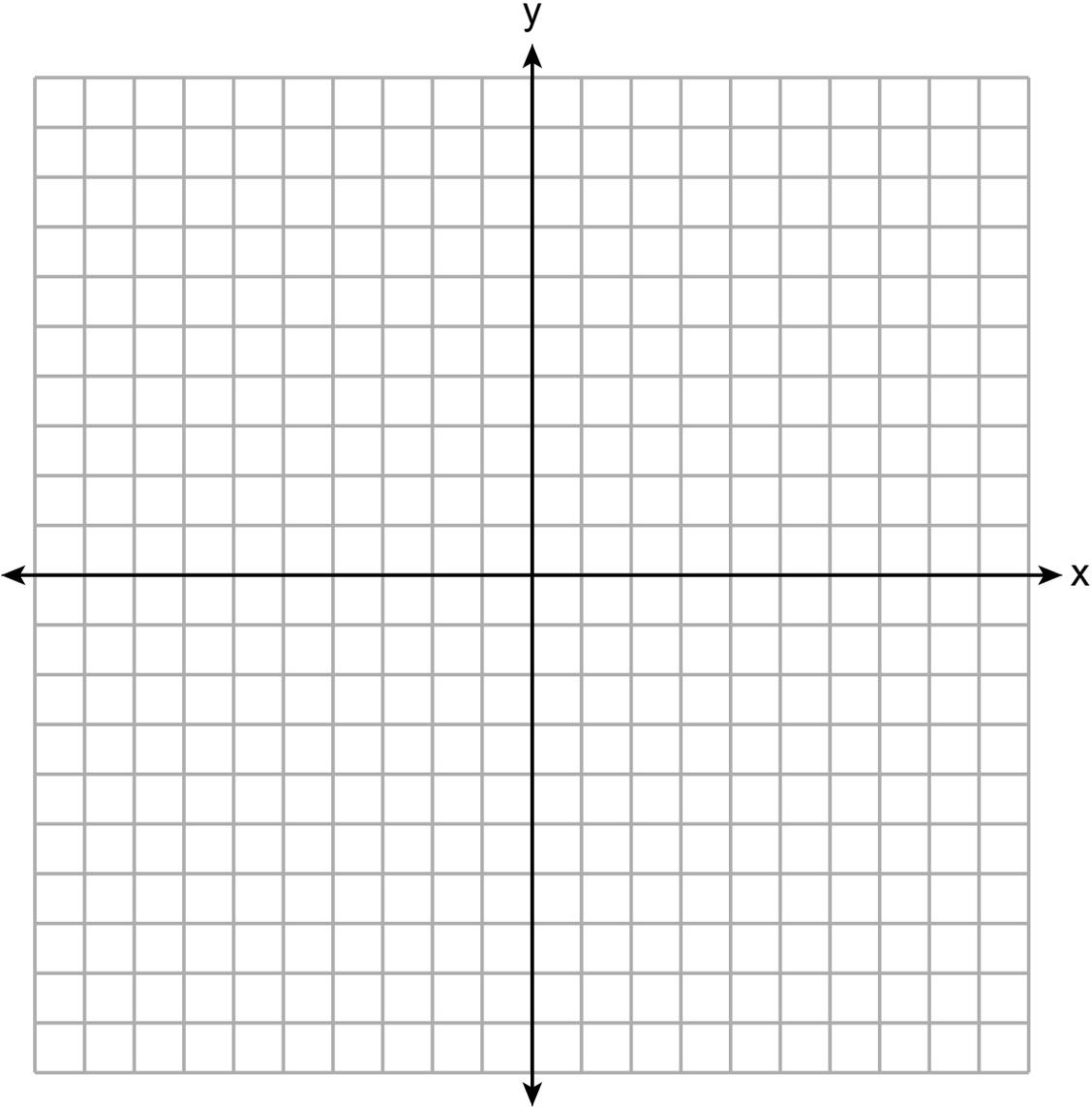
**Question 35 continued**

Point  $Y(2,2)$  is on  $\overline{OE}$ .

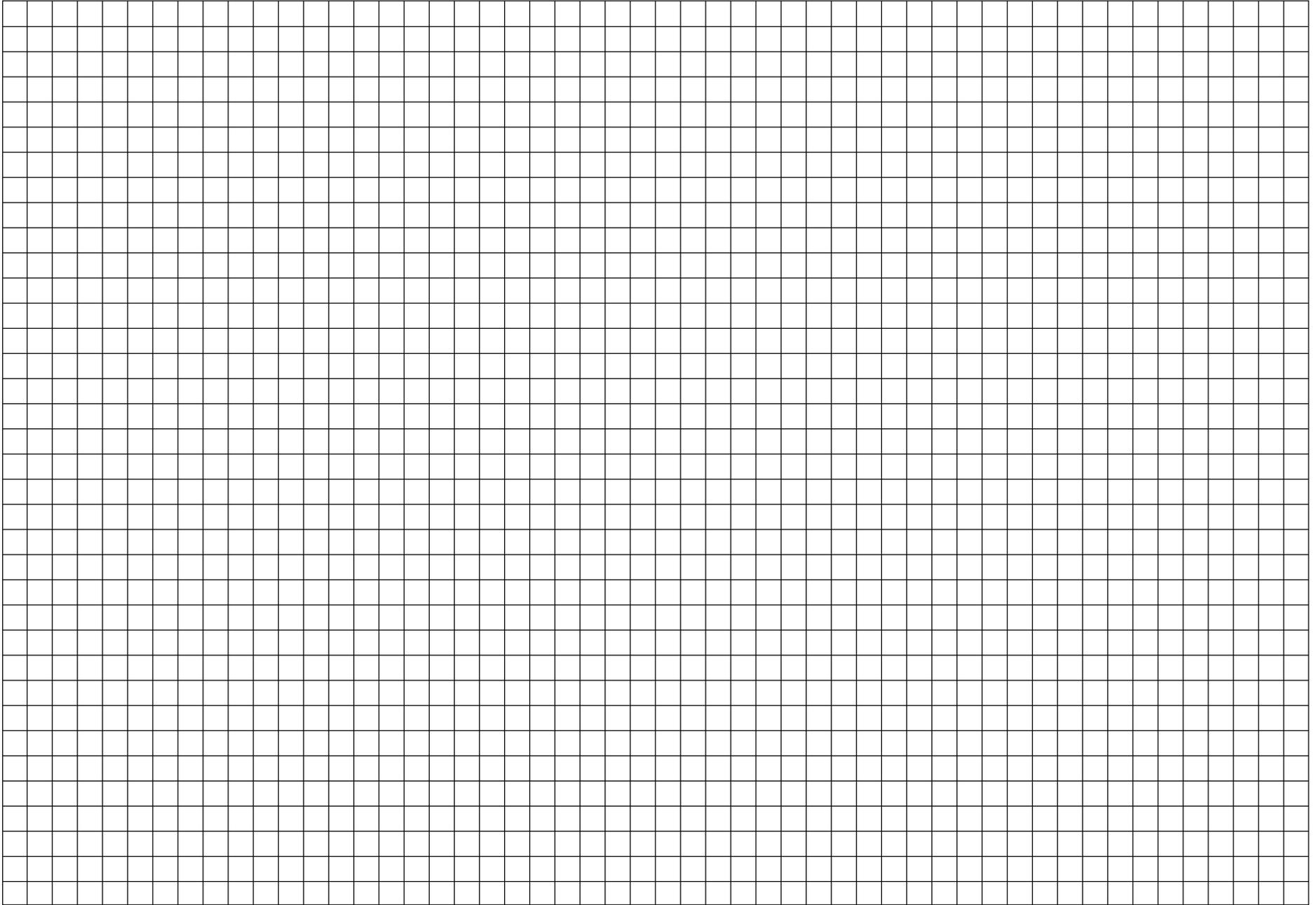
Prove that  $\overline{JY}$  is the perpendicular bisector of  $\overline{OE}$ .

**The set of axes for question 35 is on the next page.**

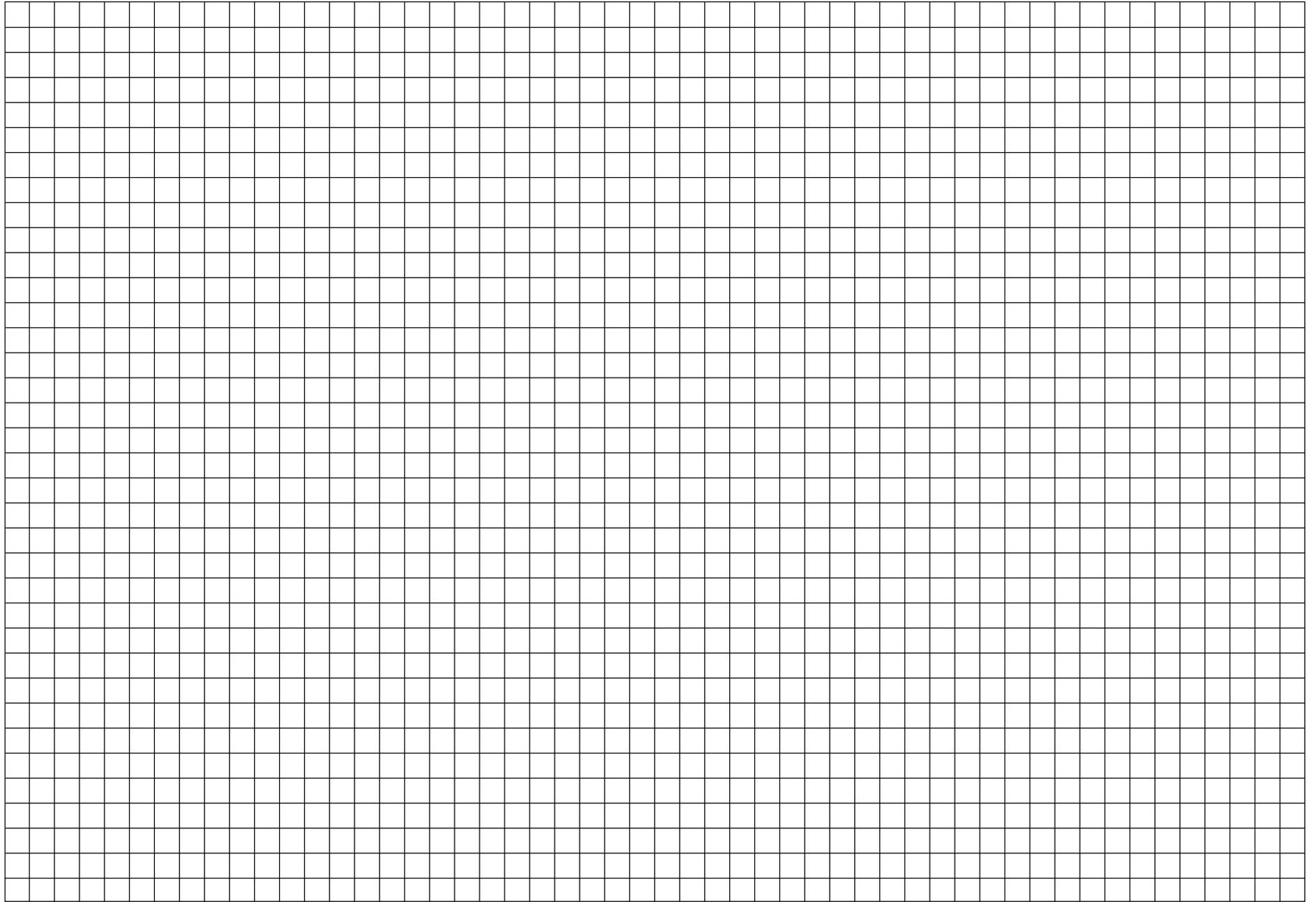
**Question 35 continued**



Scrap Graph Paper — this sheet will *not* be scored.



Scrap Graph Paper — this sheet will *not* be scored.



## High School Math Reference Sheet

1 inch = 2.54 centimeters

1 meter = 39.37 inches

1 mile = 5280 feet

1 mile = 1760 yards

1 mile = 1.609 kilometers

1 kilometer = 0.62 mile

1 pound = 16 ounces

1 pound = 0.454 kilogram

1 kilogram = 2.2 pounds

1 ton = 2000 pounds

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

1 gallon = 3.785 liters

1 liter = 0.264 gallon

1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n - 1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

**The Reference Sheet is continued on the next page.**

## Reference Sheet — concluded

Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$