ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, January 26, 2023 — 1:15 to 4:15 p.m., only

Student Name _____

School Name _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Which expression is equivalent to $(x + 2)^2 - 5(x + 2) + 6$?

(1) $x(x - 1)$

(2) $(x - 3)(x - 2)$

(3) $(x - 4)(x + 3)$

(4) $(x - 6)(x + 1)$

2 To the *nearest tenth*, the solution to the equation $4300e^{0.07x} - 123 = 5000$ is

(1) 1.1

(2) 2.5

(3) 6.3

(4) 68.5

3 The value of an automobile t years after it was purchased is given by the function $V = 38{,}000(0.84)^t$. Which statement is true?

(1) The value of the car increases 84% each year.

(2) The value of the car decreases 84% each year.

(3) The value of the car increases 16% each year.

(4) The value of the car decreases 16% each year.

4 Which function represents exponential decay?

(1) $p(x) = \left(\frac{1}{4}\right)^{-x}$

(2) $q(x) = 1.8^{-x}$

(3) $r(x) = 2.3^{2x}$

(4) $s(x) = 4^{\frac{x}{2}}$

Use this space for computations.

5 The expression $\frac{x^4 - 5x^2 + 4x + 14}{x + 2}$ is equivalent to

(1) $x^3 - 2x^2 - x + 6 + \frac{2}{x + 2}$

(2) $x^3 - 5x + 4 - \frac{14}{x + 2}$

(3) $x^3 + 2x^2 - x + 2 + \frac{18}{x + 2}$

(4) $x^3 + 2x^2 - 9x + 22 - \frac{30}{x + 2}$

6 The sum of the first 20 terms of the series $-2 + 6 - 18 + 54 - \ldots$ is

(1) -610

(2) -59

(3) 1,743,392,200

(4) 2,324,522,934

7 If $f(x) = 2x^4 - x^3 - 16x + 8$, then $f\left(\frac{1}{2}\right)$

(1) equals 0 and $2x + 1$ is a factor of $f(x)$

(2) equals 0 and $2x - 1$ is a factor of $f(x)$

(3) does not equal 0 and $2x + 1$ is not a factor of $f(x)$

(4) does not equal 0 and $2x - 1$ is a factor of $f(x)$

8 If $(6 - ki)^2 = 27 - 36i$, the value of k is

(1) -36

(2) -3

(3) 3

(4) 6

Use this space for computations.

9 What is the solution set of the equation $\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2+6}{3x}$?

(1) $\{-3\}$
(2) $\{-3,0\}$
(3) $\{3\}$
(4) $\{0,3\}$

10 How many real solutions exist for the system of equations below?

$$y = \frac{1}{4}x - 8$$
$$y = \frac{1}{2}x^2 + 2x$$

(1) 1
(2) 2
(3) 3
(4) 0

11 Which equation represents a polynomial identity?

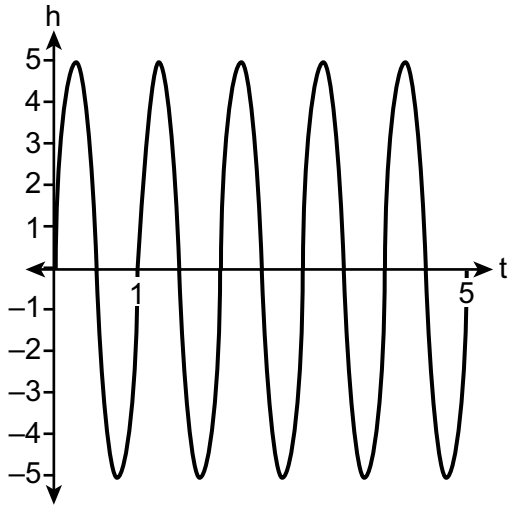
(1) $x^3 + y^3 = (x+y)^3$
(2) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
(3) $x^3 + y^3 = (x+y)(x^2 - xy - y^2)$
(4) $x^3 + y^3 = (x-y)(x^2 + xy + y^2)$

12 Given $x > 0$, the expression $\frac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}}$ can be rewritten as

(1) $\sqrt[3]{x}$
(2) $-\sqrt[10]{x^3}$
(3) $\frac{1}{\sqrt[10]{x^3}}$
(4) $\sqrt[3]{x^{10}}$

Use this space for computations.

13 A cyclist pedals a bike at a rate of 60 revolutions per minute. The height, h, of a pedal at time t, in seconds, is plotted below.



The graph can be modeled by the function $h(t) = 5\sin(kt)$, where k is equal to

(1) 1

(2) 2π

(3) 60

(4) $\frac{\pi}{30}$

14 Which statement about data collection is most accurate?

(1) A survey about parenting styles given to every tenth student entering the library will provide unbiased results.

(2) An observational study allows a researcher to determine the cause of an outcome.

(3) Margin of error increases as sample size increases.

(4) A survey collected from a random sample of students in a school can be used to represent the opinions of the school population.

Use this space for computations.

15 If $f(x) = \frac{1}{2}x + 2$, then the inverse function is

(1) $f^{-1}(x) = -\frac{1}{2}x - 2$

(2) $f^{-1}(x) = \frac{1}{2}x - 1$

(3) $f^{-1}(x) = 2x - 4$

(4) $f^{-1}(x) = 2x + 2$

16 Given $f(x) = x^4 - x^3 - 6x^2$, for what values of x will $f(x) > 0$?

(1) $x < -2$, only

(2) $x < -2$ or $x > 3$

(3) $x < -2$ or $0 \leq x \leq 3$

(4) $x > 3$, only

17 For which approximate value(s) of x will $\log(x + 5) = |x - 1| - 3$?

(1) 5, 1

(2) -2.41, 0.41

(3) -2.41, 5

(4) 5, only

18 Consider a cubic polynomial with the characteristics below.

- exactly one real root
- as $x \to \infty, f(x) \to -\infty$

Given $a > 0$ and $b > 0$, which equation represents a cubic polynomial with these characteristics?

(1) $f(x) = (x - a)(x^2 + b)$

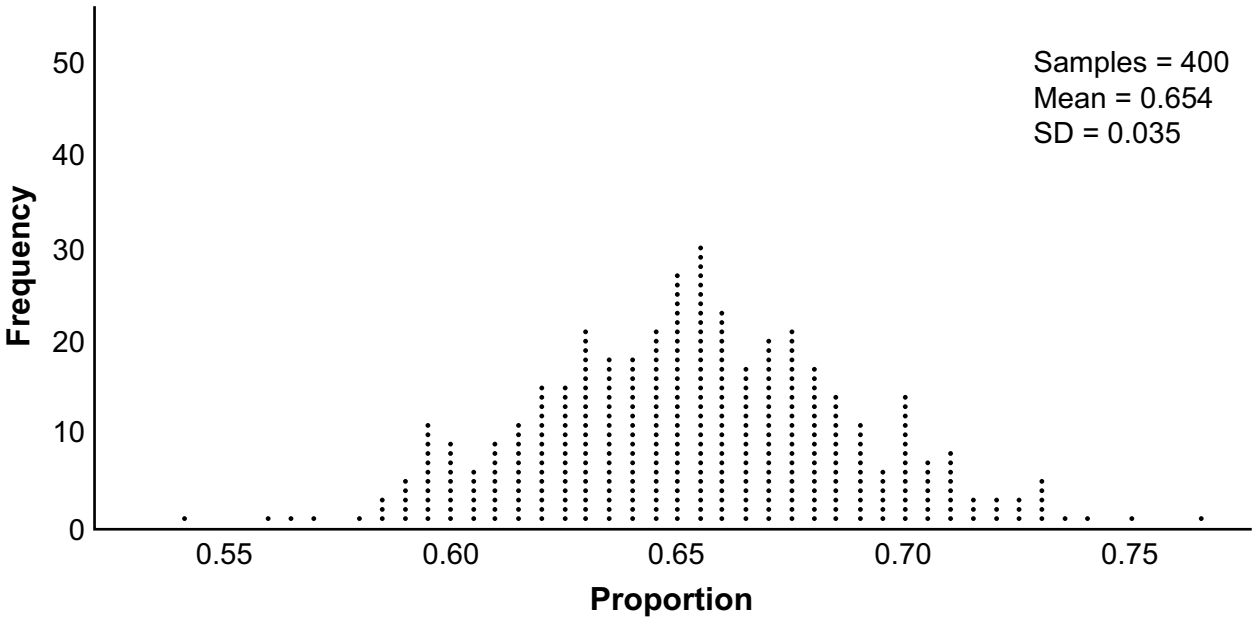
(2) $f(x) = (a - x)(x^2 + b)$

(3) $f(x) = (a - x^2)(x^2 + b)$

(4) $f(x) = (x - a)(b - x^2)$

Use this space for computations.

19 Betty conducted a survey of her class to see if they like pizza. She gathered 200 responses and 65% of the voters said they did like pizza. Betty then ran a simulation of 400 more surveys, each with 200 responses, assuming that 65% of the voters would like pizza. The output of the simulation is shown below.



Considering the middle 95% of the data, what is the margin of error for the simulation?

(1) 0.01
(2) 0.02
(3) 0.05
(4) 0.07

20 If $\cos A = \frac{\sqrt{5}}{3}$ and $\tan A < 0$, what is the value of $\sin A$?

(1) $\frac{2}{3}$
(2) $-\frac{\sqrt{5}}{3}$
(3) $-\frac{2}{3}$
(4) $\frac{3}{\sqrt{5}}$

Use this space for computations.

21 A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees, a_n, after n years?

(1) $a_1 = 150$
$a_n = a_{n-1}(0.2) + 80$

(2) $a_1 = 150$
$a_n = a_{n-1}(0.8) + 80$

(3) $a_n = 150(0.2)^n + 80$

(4) $a_n = 150(0.8)^n + 80$

22 Which equation represents a parabola with a focus of $(4,-3)$ and directrix of $y = 1$?

(1) $(x - 1)^2 = 4(y + 3)$

(2) $(x - 1)^2 = -8(y - 3)$

(3) $(x + 4)^2 = 4(y - 3)$

(4) $(x - 4)^2 = -8(y + 1)$

23 Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function $f(x) = 35{,}000(1.0325)^x$, where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be

(1) $f(x) = 35{,}000(1.0027)^{12x}$

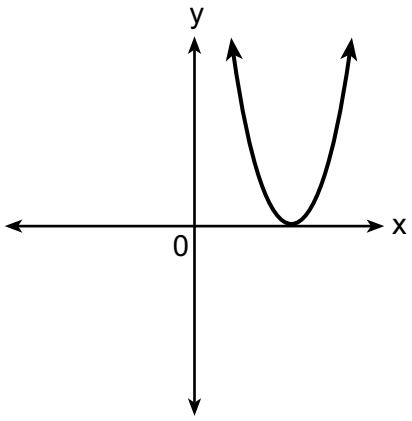
(2) $f(x) = 35{,}000(1.0027)^{\frac{x}{12}}$

(3) $f(x) = 35{,}000(1.0325)^{12x}$

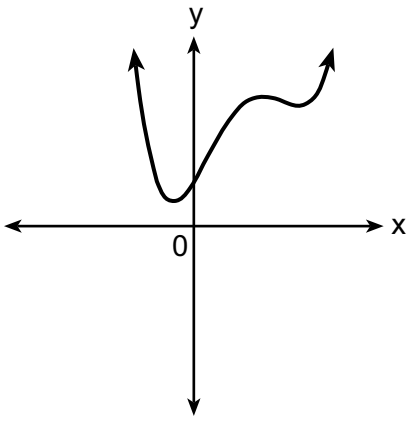
(4) $f(x) = 35{,}000(1.0325)^{\frac{x}{12}}$

Use this space for computations.

24 Which graph shows a quadratic function with two imaginary zeros?



(1)



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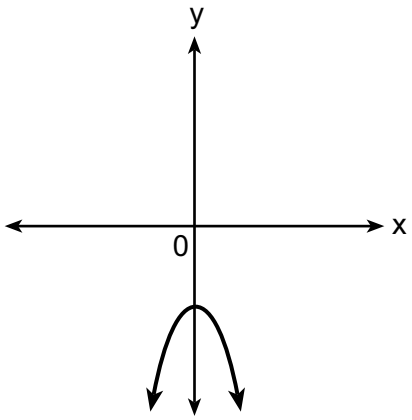
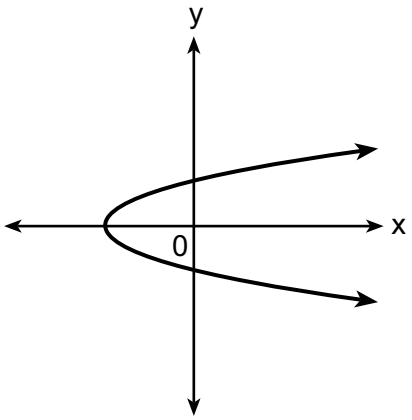


(2)



(4)

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Algebraically determine the zeros of the function below.

$$r(x) = 3x^3 + 12x^2 - 3x - 12$$

26 Given $a > 0$, solve the equation $a^{x+1} = \sqrt[3]{a^2}$ for x algebraically.

27 Given $P(A) = \frac{1}{3}$ and $P(B) = \frac{5}{12}$, where A and B are independent events, determine $P(A \cap B)$.

28 The scores on a collegiate mathematics readiness assessment are approximately normally distributed with a mean of 680 and a standard deviation of 120.

Determine the percentage of scores between 690 and 900, to the *nearest percent*.

29 Consider the data in the table below.

x	1	2	3	4	5	6
y	3.9	6	11	18.1	28	40.3

State an exponential regression equation to model these data, rounding all values to the *nearest thousandth*.

30 Write the expression $A(x) \bullet B(x) - 3C(x)$ as a polynomial in standard form.

$$A(x) = x^3 + 2x - 1$$
$$B(x) = x^2 + 7$$
$$C(x) = x^4 - 5x$$

31 Over the set of integers, completely factor $x^4 - 5x^2 + 4$.

32 Natalia's teacher has given her the following information about angle θ.

- $\pi < \theta < 2\pi$
- $\cos \theta = \frac{\sqrt{3}}{4}$

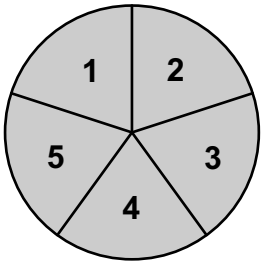
Explain how Natalia can determine if the value of $\tan \theta$ is positive or negative.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the equation $\sqrt{49 - 10x} + 5 = 2x$ algebraically.

34 Joette is playing a carnival game. To win a prize, one has to correctly guess which of five equally sized regions a spinner will land on, as shown in the diagram below.



She complains that the game is unfair because her favorite number, 2, has only been spun once in ten times she played the game.

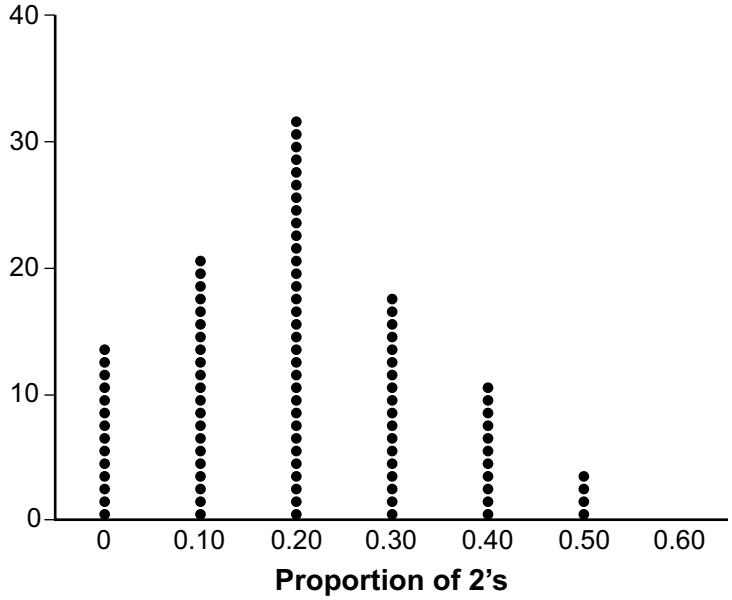
State the proportion of 2's that were spun.

State the theoretical probability of spinning a 2.

Question 34 is continued on the next page.

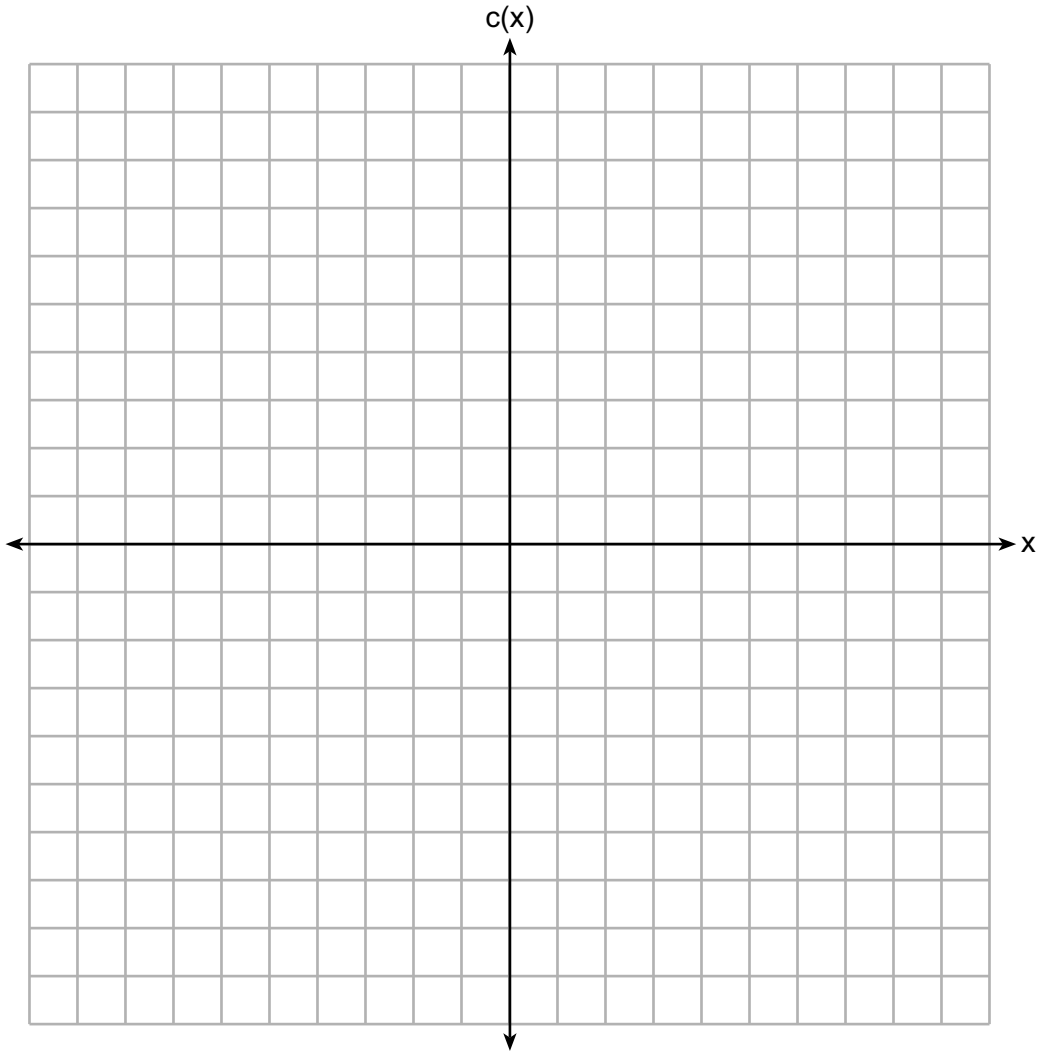
Question 34 is continued

The simulation output below shows the results of simulating ten spins of a fair spinner, repeated 100 times.



Does the output indicate that the carnival game was unfair? Explain your answer.

35 Graph $c(x) = -9(3)^{x-4} + 2$ on the axes below.



Describe the end behavior of $c(x)$ as x approaches positive infinity.

Describe the end behavior of $c(x)$ as x approaches negative infinity.

36 The monthly high temperature (°F) in Buffalo, New York can be modeled by $B(m) = 24.9\sin(0.5m - 2.05) + 55.25$, where m is the number of the month and January = 1.

Find the average rate of change in the monthly high temperature between June and October, to the *nearest hundredth*.

Explain what this value represents in the given context.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Objects cool at different rates based on the formula below.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

T_0: initial temperature
T_R: room temperature
r: rate of cooling of the object
t: time in minutes that the object cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Using this information, write an equation for the temperature of the shirt, T, after t minutes.

Use the equation to find the temperature of the shirt, to the *nearest degree*, after five minutes.

Question 37 is continued on the next page.

Question 37 is continued

At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*.

The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

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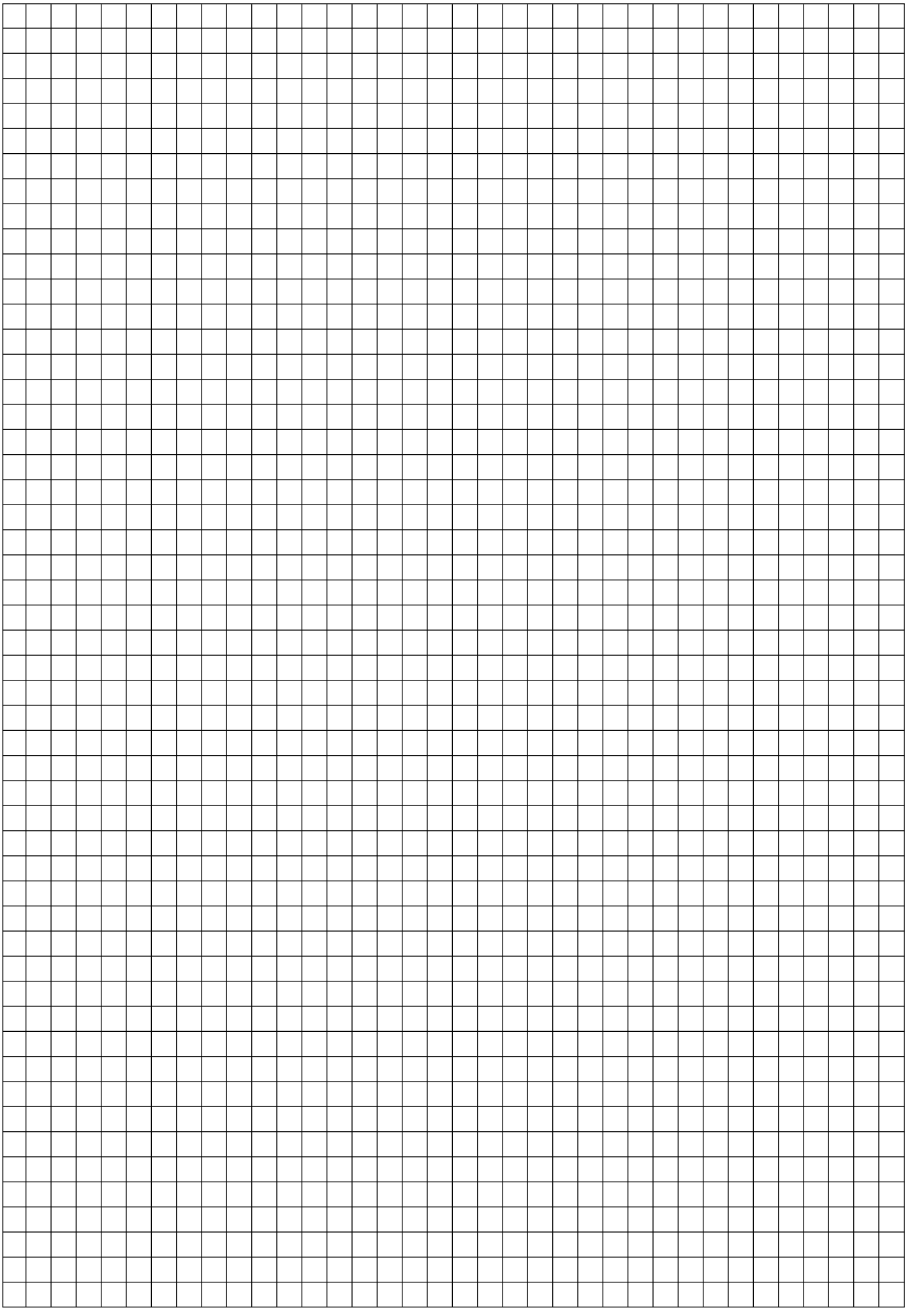

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High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - \mathrm{r}}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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