THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, January 29, 2004 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 A + 1 = \sec^2 A$$

$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2} A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2} A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2} ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 What is the amplitude of the graph of the equation $y = 4 \sin \frac{1}{2}x$?
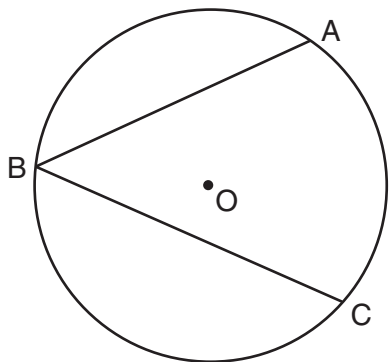
2 Express as a monomial in terms of i:
$$8\sqrt{-36} - 4\sqrt{-49}$$

3 If m$\angle A$ = 30, side b = 8, and side c = 4, find the area of $\triangle ABC$.

4 If $3x$ is the measure of a positive acute angle and $\cos 3x = \sin 60°$, find the value of x.

5 Find the image of $P(2,-5)$ under the transformation $r_{y=x}$.

6 In the accompanying diagram of circle O, m$\angle ABC$ = $2x$ and m$\overset{\frown}{AC}$ = x + 60. Find the value of x.

7 What is the value of $3a^0 + a^{\frac{1}{2}} + 8a^{-2}$ when $a = 4$?

8 If f(x) = $\sin x + \cos \frac{x}{2}$, find f($2\pi$).

9 Solve for x: $16^{x+4} = 32^{2x-10}$

10 Solve for x: $\frac{2}{3x} + 5 = \frac{4}{x}$

11 In $\triangle ABC$, side a = 3, side c = $3\sqrt{2}$, and m$\angle A$ = 45. Find m$\angle C$.

12 Find a value for θ in the interval $90° \le \theta \le 270°$ that satisfies the equation $2 \sin \theta + 1 = 0$.

13 Express the value of $\sin \left(\text{Arc} \tan \sqrt{3}\right)$ in simplest radical form.

14 Evaluate: $\frac{1}{3} \sum_{k=2}^{4} \left| k - 5 \right|$

15 In a circle whose radius is 10, what is the length of the arc intercepted by a central angle of 4 radians?

16 In the accompanying diagram, tangent $\overline{AB}$ and secant $\overline{ACD}$ are drawn to circle O from point A, AB = 6, and AC = 4. Find AD.
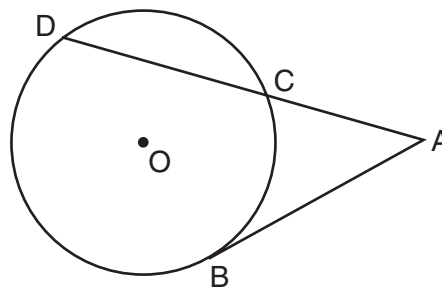
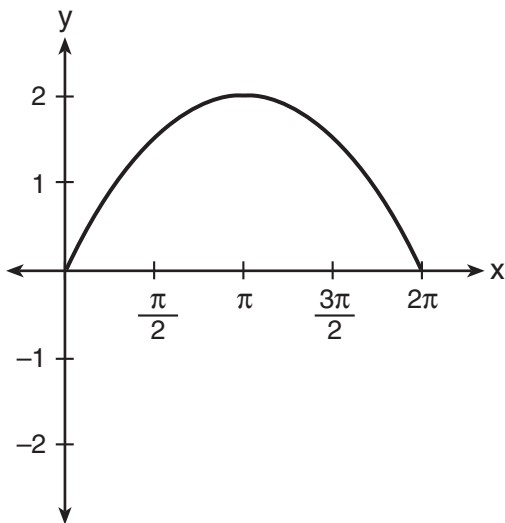17 If x varies inversely as y, and x = -34 when y = -2, find x when y = 4.

Directions (18–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 The expression $\sin 240°$ is equivalent to
(1) $\sin 60°$ (3) $-\sin 60°$
(2) $\cos 60°$ (4) $-\cos 60°$

19 Which equation is represented by the accompanying graph?

(1) $y = 2 \sin \frac{1}{2}x$

(2) $y = 2 \sin x$

(3) $y = \sin \frac{1}{2}x$

(4) $y = \sin 2x$

20 For which value of x is the expression $\frac{\sin x}{1 + 2\cos x}$ undefined?

(1) 60°

(2) 120°

(3) 180°

(4) 300°

21 The probability that Laura wins a tennis match against Jennifer is $\frac{2}{3}$. What is the probability that Laura wins *exactly* three of the next four matches she plays against Jennifer?

(1) $\frac{32}{81}$

(2) $\frac{8}{81}$

(3) $\frac{108}{256}$

(4) $\frac{27}{256}$

22 When the sum of the complex numbers $3 + 2i$ and $6 - 4i$ is graphed, in which quadrant does it lie?

(1) I

(2) II

(3) III

(4) IV

23 What is the solution set of $|4x + 8| > 16$?

(1) $\{x \mid -6 < x < 2\}$

(2) $\{x \mid -2 < x < 6\}$

(3) $\{x \mid x < -6 \text{ or } x > 2\}$

(4) $\{x \mid x < -2 \text{ or } x > 6\}$

24 In the set of real numbers, what is the domain of $f(x) = \frac{4x}{\sqrt{x - 4}}$?

(1) $x > 0$

(2) $x < 4$

(3) $x \geq 4$

(4) $x > 4$

25 What is the fifth term in the expansion of $(x - 2)^6$?

(1) $_6C_2\, x^4(-2)^2$

(2) $_6C_5\, x^5(-2)^1$

(3) $_6C_1\, x(-2)^5$

(4) $_6C_4\, x^2(-2)^4$

26 The heights of the girls in the eleventh grade are normally distributed with a mean of 66 inches and a standard deviation of 2.5 inches. In which interval do approximately 95% of the heights fall?

(1) 61–66 inches

(2) 61–71 inches

(3) 63.5–68.5 inches

(4) 66–71 inches

27 The expression $(\sec^2 \theta)(\cot^2 \theta)(\sin \theta)$ is equivalent to

(1) $\sin \theta$

(2) $\cos \theta$

(3) $\csc \theta$

(4) $\sec \theta$

28 The expression $\dfrac{x + \frac{x}{y}}{1 + \frac{1}{y}}$ is equivalent to

(1) x

(2) $2x$

(3) $\frac{x}{y}$

(4) $x + 1$

29 What is the sum of the roots of the equation $2x^2 - 13x + 17 = 0$?

(1) $-\frac{13}{2}$

(2) $\frac{13}{2}$

(3) $-\frac{17}{2}$

(4) $\frac{17}{2}$

30 What is the solution set of the inequality $-2x^2 + 3x + 5 > 0$?

(1) $\{x \mid -1 < x < 2.5\}$

(2) $\{x \mid -2.5 < x < 1\}$

(3) $\{x \mid x < -1 \text{ or } x > 2.5\}$

(4) $\{x \mid x < -2.5 \text{ or } x > 1\}$

31 Which equation, when graphed, is an ellipse?

(1) $2x^2 + 4 = 2y^2$

(2) $xy = 6$

(3) $4x^2 + 4y^2 = 25$

(4) $7x^2 + 3y^2 = 9$

32 If $\sec x < 0$ and $\cot x < 0$, in which quadrant does the terminal side of angle x lie?

(1) I   (3) III

(2) II   (4) IV

33 If $m\angle A = 68$, side $a = 10$, and side $b = 24$, how many distinct triangles can be constructed?

(1) 1   (3) 3

(2) 2   (4) 0

34 Which equation is the inverse of $y = 13x + 2$?

(1) $y = 2x + 13$ (3) $y = \dfrac{x - 2}{13}$

(2) $y = -13x - 2$ (4) $y = \dfrac{x - 13}{2}$

35 What is the image of point $(8,-4)$ under the rotation $R_{90°}$ about the origin?

(1) $(8,4)$ (3) $(-4,8)$

(2) $(4,8)$ (4) $(-4,-8)$

Answers to the following questions are to be written on paper provided by the school.

Part II

    Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.   [40]

36 In the accompanying diagram, $\triangle ABC$ is inscribed in circle O, $\overline{AP}$ bisects $\angle BAC$, $\overrightarrow{PBD}$ is tangent to circle O at B, and $m\angle ACB{:}m\angle CAB{:}m\angle ABC = 4{:}3{:}2$.
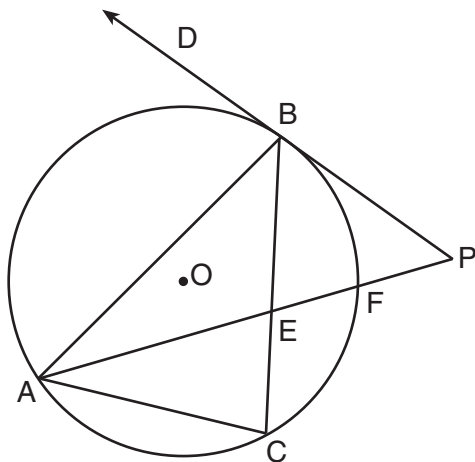
Find:

a $m\angle ABC$ [2]

b $m\,\widehat{BF}$ [2]

c $m\angle BEP$ [2]

d $m\angle P$ [2]

e $m\angle PBC$ [2]

37 Find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $5 \sin \theta + 2 \cos 2\theta - 3 = 0$. Express your answer to the *nearest ten minutes* or *nearest tenth of a degree*.   [10]

38 *a*   Express in simplest form:

$$\frac{x^2 - 16}{2x^2 + 4x} \cdot \frac{x^2 + 9x + 14}{x^2 + 2x - 8} \div \frac{x^2 + 3x - 28}{16x - 8x^2} \quad [6]$$

  b   Solve the equation for x and express the roots in simplest $a + bi$ form:

$$3x^2 = 2x - 1 \quad [4]$$

39 *a*   Solve for all values of x:

$$\log_3 (x + 4) + \log_3 (x - 2) = 3 \quad [6]$$

  b   Given: $\log_b 2 = x$ and $\log_b 3 = y$

      Express in terms of x and y:

      (1) $\log_b 18$ [2]

      (2) $\log_b \frac{9}{16}$ [2]

40 *a*   On the same set of axes, sketch and label the graphs of the equations $y = 2 \cos 2x$ and $y = -\sin x$ in the interval $-\pi \le x \le \pi$.   [8]

  b   Based on the graphs drawn in part *a*, determine the number of values of x in the interval $-\pi \le x \le \pi$ that satisfy the equation $2 \cos 2x = -\sin x$.   [2]

41 Mrs. Gruber gave her history class a multiple-choice quiz containing five questions. A student must answer at least four questions correctly to pass.

a Greg decided to guess on every question. If each of the four possible answers to each question is equally likely to be chosen, what is the probability that Greg passed the quiz? [4]

b The scores for the 20 students in the class are shown in the accompanying table.

Score	0	20	40	60	80	100
Frequency	3	1	2	4	8	2

(1) Find the mean of this set of data. [1]
(2) Find the standard deviation of this set of data to the *nearest tenth*. [3]
(3) What percent of the scores fell within one standard deviation of the mean? [2]

42 As shown in the accompanying diagram, a ship at sea is 75 miles from radio transmitter A and 120 miles from radio transmitter B. The angle between the signals sent to the ship by the two transmitters measures 135°.
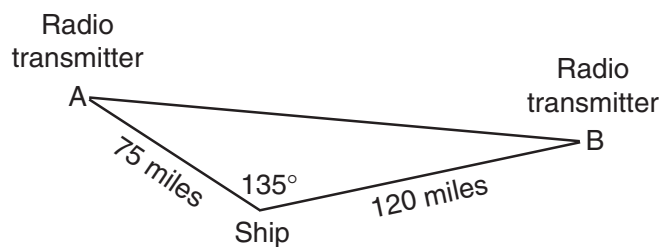
a Find the distance between the transmitters to the *nearest mile*. [6]

b Using the answer found in part a, find the measure of angle B to the *nearest degree*. [4]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Thursday, January 29, 2004 — 9:15 a.m. to 12:15 p.m., only

Part I Score	. . . .
Part II Score	. . . .
Total Score	. . . .
Rater's Initials:	. . . .

ANSWER SHEET

Student . . . . Sex: ☐ Male ☐ Female   Grade . . . .

Teacher . . . . School . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

[8]