The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Friday, June 20, 2003 — 1:15 to 4:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 Evaluate: $\sum_{x=1}^{3}(2x+1)$

2 If $f(b) = b^0 + b^{-1} + b^{-2}$, find f(2).

3 Solve for x: $\sqrt{2x-4} - 6 = 0$

4 In $\triangle ABC$, side $a = 18$, $\sin A = \frac{3}{4}$, and $\sin B = \frac{2}{3}$. Find the length of side b.

5 If $f(x) = 3x + 2$ and $g(x) = x^2 - 5$, find the value of $(f \circ g)(-3)$.

6 In $\triangle ABC$, $m\angle B = 30$ and side $a = 6$. If the area of the triangle is 12, what is the length of side c?

7 Find the image of point $A(3,-2)$ under the composition of translations $T_{2,1} \circ T_{-6,-4}$.

8 For what values of x is the fraction $\frac{4-x}{x^2-4}$ undefined?

9 Express $(1 - \cos\theta)(1 + \cos\theta)$ in terms of $\sin\theta$.

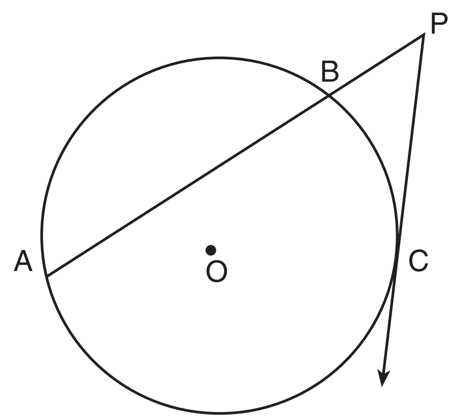
10 If $\sin(2x + 20)° = \cos 40°$, find x.

11 Find a positive acute angle θ such that $4 \cot\theta \sin\theta = 2$.

12 An angle of $2\frac{1}{4}$ radians at the center of a circle intercepts an arc of 18 inches. Find the length of the radius in inches.

13 What is the solution set of the equation $|2x + 5| - 4 = 3$?

14 In the accompanying diagram, $\overrightarrow{PC}$ is tangent to circle O, $\overline{PBA}$ is a secant, $PC = 6$, and $PB = 3$. Find AB.



15 If x varies inversely as y, and $x = 10$ when $y = 12$, what is the value of x when $y = 8$?

16 In $\triangle DEF$, if side $d = 14$, side $e = 10$, and side $f = 12$, find $m\angle F$ to the *nearest degree*.

17 What are the coordinates of the image of point $A(3,-1)$ after a reflection in the line $x = 2$?

Directions (18–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

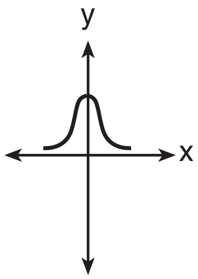
18 What are the coordinates of point $(-1,4)$ under dilation D_{-2}?

(1) $(-2,8)$
(2) $(2,-8)$
(3) $(-8,2)$
(4) $(8,-2)$

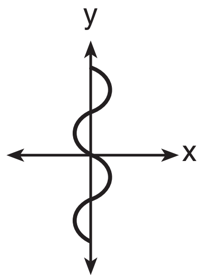
19 The expression $\log \frac{b^3}{a}$ is equivalent to

(1) $3(\log b - \log a)$
(2) $\log 3b - \log a$
(3) $3 \log b - \log a$
(4) $\frac{3 \log b}{\log a}$

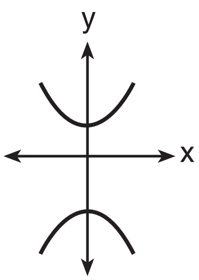
20 Which graph represents a function?



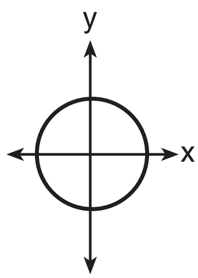
(1)



(3)



(2)



(4)

21 If $\sin B < 0$ and $\cos B > 0$, in which quadrant does angle B terminate?

(1) I (3) III
(2) II (4) IV

22 What is the value of $\tan\left(\text{Arc}\cos\frac{5}{13}\right)$?

(1) $\frac{12}{13}$ (3) $\frac{12}{5}$
(2) $\frac{5}{12}$ (4) $\frac{13}{5}$

23 On a mathematics quiz with a normal distribution, the mean is 8. If the standard deviation is 0.5, what is the best approximation of the percentage of grades that lie between 7 and 9?

(1) 5% (3) 68%
(2) 34% (4) 95%

24 What is the range of the function $y = 4\cos x$?

(1) $-1 \le y \le 1$ (3) $y \ge 0$
(2) $-4 \le y \le 4$ (4) $y \le 4$

25 If one root of the equation $x^2 + kx - 15 = 0$ is -3, what is the other root?

(1) -2 (3) 3
(2) 2 (4) 5

26 In which quadrant does the graph of $y = \left(\frac{1}{2}\right)^x$ intersect the graph of $y = x$?

(1) I (3) III
(2) II (4) IV

27 The reciprocal of the expression $\frac{2}{x} + \frac{3}{1}$ is

(1) $\frac{2+3x}{x}$ (3) $2x + 3$
(2) $\frac{x}{2+3x}$ (4) $2 + 3x$

28 The product of $6x^a$ and x is

(1) $6x^a$ (3) $6x^{a^2}$
(2) $6x^{a+1}$ (4) $6x^{2a}$

29 What is the third term in the expansion of $(\cos x - 1)^4$?

(1) $6\cos^2 x$ (3) $4\cos x$
(2) $-6\cos^2 x$ (4) $-4\cos x$

30 The expression $\sin(180° + x)$ is equivalent to

(1) $\sin x$ (3) $-\sin x$
(2) $\cos x$ (4) $-\cos x$

31 If i is the imaginary unit, the expression $i^8 + i^9 + i^{10} + i^{11}$ is equivalent to

(1) 1 (3) i
(2) -1 (4) 0

32 Which inequality is represented by the graph below?

(1) $x^2 - 2x - 15 > 0$ (3) $x^2 - 2x - 15 \le 0$
(2) $x^2 - 2x - 15 < 0$ (4) $x^2 + 2x - 15 < 0$

33 The expression $\frac{7}{2+3\sqrt{2}}$ is equivalent to

(1) $\frac{-2+3\sqrt{2}}{2}$ (3) $-2 + 3\sqrt{2}$
(2) $\frac{2-3\sqrt{2}}{2}$ (4) $2 - 3\sqrt{2}$

34 The roots of the equation $3x^2 - 4x - 5 = 0$ are

(1) real, rational, and equal
(2) real, rational, and unequal
(3) real, irrational, and unequal
(4) imaginary

35 If m$\angle A = 50$, side $a = 6$, and side $b = 10$, what is the maximum number of distinct triangles that can be constructed?

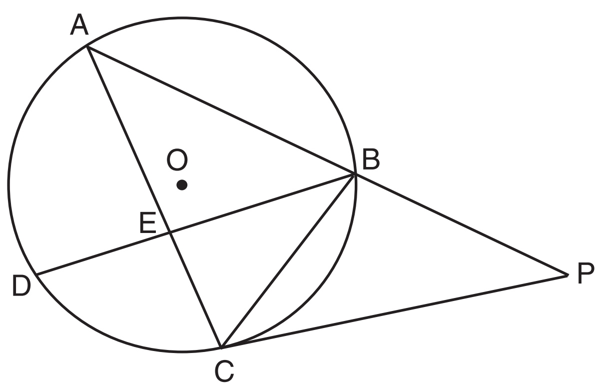
(1) 1
(2) 2
(3) 3
(4) 0

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 In the accompanying diagram of circle O, chords $\overline{BD}$, $\overline{BC}$, and $\overline{AC}$, tangent $\overline{PC}$, and secant $\overline{ABP}$ are drawn; m$\angle DBC = 40$; m$\angle AEB = 110$; and m$\widehat{AD}$:m$\widehat{CB} = 9:5$.



Find:

a m$\widehat{AB}$ [2]
b m$\widehat{AD}$ [2]
c m$\angle P$ [2]
d m$\angle BCP$ [2]
e m$\angle ACP$ [2]

37 Find, to the *nearest ten minutes* or *nearest tenth of a degree,* all values of x in the interval $0° \le x < 360°$ that satisfy the equation $4 \cos 2x - 2 \cos x + 3 = 0$. [10]

38 *a* On the same set of axes, sketch and label the graphs of the equations $y = \frac{1}{2} \sin 2x$ and $y = -2 \cos x$ in the interval $0 \le x \le 2\pi$. [8]

b Using the graphs drawn in part *a*, find all values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $\frac{1}{2} \sin 2x = -2 \cos x$. [2]

39 *a* Prove the following identity:
$\tan x + \cot x = \csc x \sec x$ [6]

b Given: $\log 2 = x$ and $\log 11 = y$

Express in terms of x and y:

(1) $\log \sqrt[3]{\frac{2}{11}}$ [2]

(2) $\log 44$ [2]

40 *a* Five cards are in a box. Two are red and three are black. Four cards are selected at random and replaced in the box after each selection.

(1) Find the probability that *exactly* three of the cards selected are black. [2]

(2) Find the probability of selecting *at most* one red card. [2]

b Hotels are rated on the basis of one star to five stars. The accompanying table represents the ratings of 50 hotels.

Number of Stars (x_i)	**Frequency** (f_i)
1	7
2	10
3	22
4	8
5	3

(1) Find the standard deviation of this set of data to the *nearest hundredth.* [4]

(2) How many of the hotels have ratings that fall within one standard deviation of the mean? [2]

41 In $\triangle ABC$, side $a = 13$, side $b = 25$, and $m\angle C = 53.8$.

a Find the length of side c to the *nearest tenth.* [6]

b Using the answer found in part *a*, find $m\angle A$ to the *nearest degree.* [4]

42 *a* Express the roots of the equation $x^2 + 1 = 4(x - 1)$ in simplest $a + bi$ form. [5]

b Solve for x: $\frac{x}{x-5} - \frac{2}{x+5} = \frac{50}{x^2-25}$ [5]

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REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Friday, June 20, 2003 — 1:15 to 4:15 p.m., only

Part I Score	. . . .
Part II Score	. . . .
Total Score	. . . .
Rater's Initials:	. . . .

ANSWER SHEET

Student . . . . Sex: ☐ Male ☐ Female Grade . . . .

Teacher . . . . School . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . .
2	12	22	32 . . . .
3	13	23	33 . . . .
4	14	24	34 . . . .
5	15	25	35 . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

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Signature

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