The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, January 30, 2003 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 What is the *negative* value of x that satisfies the equation $|3x + 1| = 8$?

2 Evaluate: $\sum_{n=2}^{5}(n-2)^2$

3 If $f(x) = 2x^3 + 4x^2$, find $f(-3)$.

4 Express 240° in radian measure.

5 When the number 0.00422 is expressed in scientific notation as 4.22×10^n, what is the value of n?

6 Find the coordinates of the image of point (5,2) after a reflection in the line $y = x$.

7 Express $-3i + \frac{1}{2}\sqrt{-64}$ as a monomial in terms of i.

8 Find the coordinates of P', the image of $P(-3,4)$ under the translation $T_{4,1}$.

9 Two tangents are drawn to a circle from an external point. If the measure of the major arc is 260°, what is the measure, in degrees, of the angle formed by the two tangents?

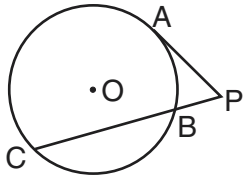
10 If $f(x) = x^{-\frac{3}{2}}$, find $f\left(\frac{16}{9}\right)$.

11 If $f(x) = x^3 + 1$ and $g(x) = x + 4$, find $(f \circ g)(-6)$.

12 If $f(x) = \frac{1}{x^2 - x}$, for which value or values of x is $f(x)$ undefined?

13 In acute triangle ABC, side $a = 10$, side $b = 12$, and $m\angle A = 42$. Find $m\angle B$ to the *nearest degree.*

14 In the accompanying diagram, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn to circle O from external point P. If $PA = 8$ and $PB = 4$, find the length of $\overline{BC}$.



15 What is the value of $\sec\left(\text{Arc cos } \frac{5}{7}\right)$?

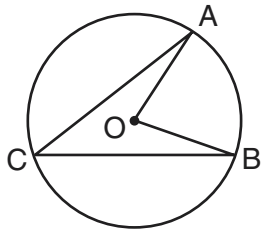
16 In a circle whose radius is 5 centimeters, a central angle intercepts an arc of 10 centimeters. What is the number of radians in the central angle?

Directions (17–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

17 The expression $\sin 80° \cos 70° + \cos 80° \sin 70°$ is equivalent to

(1) $\sin 10°$
(2) $\cos 10°$
(3) $\sin 150°$
(4) $\cos 150°$

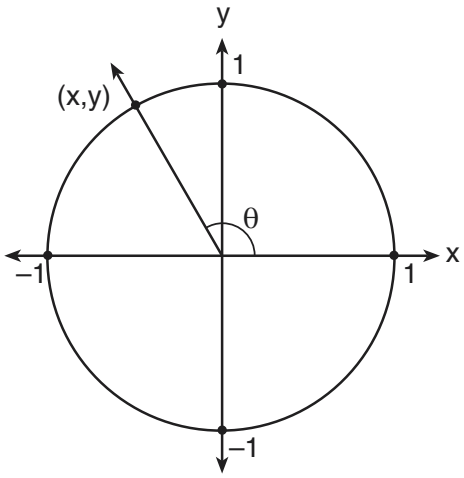
18 In the accompanying diagram of circle O, $m\angle ACB = 38$.



What is $m\angle AOB$?

(1) 19
(2) 38
(3) 52
(4) 76

19 In the accompanying diagram of a unit circle, the ordered pair (x,y) represents the point where the terminal side of θ intersects the unit circle.



If $x = -\frac{1}{2}$, what is one possible value for θ?

(1) 60° (3) 145°
(2) 120° (4) 150°

20 The expression $\dfrac{\frac{3x}{x+3}}{\frac{x}{x^2-9}}$ is equivalent to

(1) $3x - 9$ (3) $3x$
(2) $2x + 6$ (4) $\dfrac{3}{x+3}$

21 If $\sin\theta = \frac{\sqrt{7}}{4}$ and $\cos\theta = -\frac{3}{4}$, what is $\tan\theta$?

(1) $\frac{4}{3}$ (3) $\frac{\sqrt{7}}{3}$
(2) $-\frac{\sqrt{7}}{4}$ (4) $-\frac{\sqrt{7}}{3}$

22 What is the period of the function $y = -3\sin 2x$?

(1) $-\frac{\pi}{2}$ (3) 2π
(2) 2 (4) π

23 The expression $\dfrac{2\cos x}{\sin 2x}$ is equivalent to

(1) $\cos x$ (3) $\sin x$
(2) $\csc x$ (4) $\sec x$

24 If the domain is the set of real numbers, what is the solution set for the equation $x^2 + 4 = 0$?

(1) {–2} (3) {2,–2}
(2) {2} (4) { }

25 If $0° < \theta < 360°$, the solutions of the equation $9^{\sin\theta} = 3$ are 30° and

(1) 150° (3) 320°
(2) 210° (4) 330°

26 The roots of the equation $3x^2 - 7x = 5$ are

(1) real, rational, and unequal
(2) real, rational, and equal
(3) real, irrational, and unequal
(4) imaginary

27 If $a = -3 + 2i$ and $b = 4 - i$, in which quadrant does the graph of $2a - b$ lie?

(1) I (3) III
(2) II (4) IV

28 The expression $\frac{2}{\sqrt{3}+1}$ is equivalent to

(1) $\frac{\sqrt{3}}{2}$ (3) $\sqrt{3}-1$
(2) $\frac{2\sqrt{3}+2}{4}$ (4) $1-\sqrt{3}$

29 What is the solution set of $\sqrt{2-x} = x$?

(1) {1} (3) {–2,1}
(2) {–2} (4) {–1,2}

30 How many distinct triangles can be constructed if $m\angle A = 60$, side $a = 5\sqrt{3}$, and side $b = 10$?

(1) 1 (3) 3
(2) 2 (4) 0

31 The heights of a group of girls are normally distributed with a mean of 66 inches. If 95% of the heights of these girls are between 63 and 69 inches, what is the standard deviation for this group?

(1) 1 (3) 3
(2) 1.5 (4) 6

32 What is the third term in the expansion of $(a - 2b)^6$?

(1) $-160a^3b^3$ (3) $-60a^4b^2$
(2) $160a^3b^3$ (4) $60a^4b^2$

33 If $\log x = \log a - 3 \log b$, x is equal to

(1) $a - 3b$ (3) $\frac{a}{3b}$

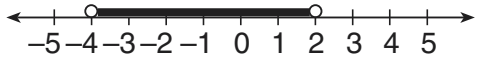
(2) $a - b^3$ (4) $\frac{a}{b^3}$

34 Which expression is equivalent to i^{233}?

(1) 1 (3) i

(2) –1 (4) $-i$

35 The graph below represents the solution set of which inequality?



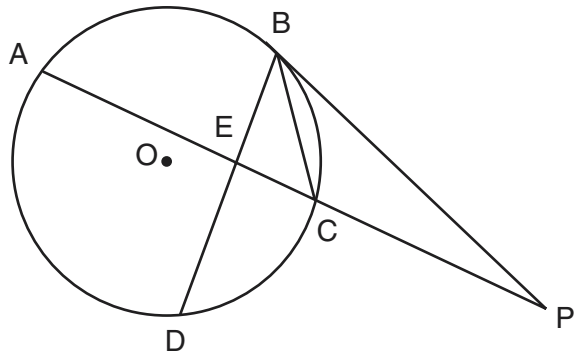
(1) $x^2 - 2x - 8 < 0$
(2) $x^2 + 2x - 8 < 0$
(3) $x^2 - 2x - 8 > 0$
(4) $x^2 + 2x - 8 > 0$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 In the accompanying diagram of circle O, tangent $\overline{PB}$, secant $\overline{AECP}$, chord $\overline{DEB}$, and chord $\overline{CB}$ are drawn; $m\overset{\frown}{DC} = 90$; $m\angle DEC = 85$; $BP = 15$; and $CB = 8$.



Find:

a $m\overset{\frown}{AB}$ [2]
b $m\angle ACB$ [2]
c $m\angle P$ to the *nearest degree* [6]

37 Find, to the *nearest tenth of a degree*, all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $5 \sin^2 \theta - 9 \cos \theta - 3 = 0$. [10]

38 *a* On the same set of axes, sketch and label the graphs of the equation $y = -3 \sin \frac{1}{2}x$ and $y = \cos 2x$ in the interval $0 \le x \le 2\pi$. [8]

b Using the graphs sketched in part *a*, determine the number of values of x in the interval $0 \le x \le 2\pi$ that satisfy the equation $\cos 2x = -3 \sin \frac{1}{2}x$. [2]

39 *a* Solve for x and express your answer in simplest $a + bi$ form:

$$x^2 + 29 = 4x \quad [5]$$

b Express in simplest form:

$$\frac{36 - x^2}{x^2 + 8x + 12} \div \frac{x^2 - 6x}{x - 2} \quad [5]$$

40 A hiking trail is planned in the shape of a triangle with sides 2.3 miles, 8.1 miles, and 6.2 miles.

a Find, to the *nearest tenth of a degree* or the *nearest ten minutes*, the angle between the 2.3-mile side and the 6.2-mile side. [6]

b Find the area of the triangle to the *nearest tenth of a square mile.* [4]

41 *a* Six golfers had the following scores for nine holes of golf: 38, 38, 43, 45, 46, and 48.

(1) Find the standard deviation of these scores to the *nearest tenth*. [4]

(2) How many scores are within one standard deviation of the mean? [1]

b A varsity basketball player makes $\frac{3}{4}$ of the foul shots she attempts. Find the probability that in four attempts she will make:

(1) *exactly* three foul shots [2]

(2) *at least* three foul shots [3]

42 *a* On graph paper, sketch the graph of the function $f(x) = \log_2 x$ in the interval $0 < x \leq 8$. [4]

b On the same set of axes, sketch the inverse of the graph drawn in part *a*, and label it *b*. [4]

c Write the equation of $f^{-1}(x)$. [2]

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SEQUENTIAL MATH – COURSE III

Thursday, January 30, 2003 — 9:15 a.m. to 12:15 p.m., only

Part I Score	. . . . .
Part II Score	. . . . .
Total Score	. . . . .
Rater's Initials:	. . . . .

ANSWER SHEET

Student . . . . . Sex: ☐ Male ☐ Female Grade . . . . .

Teacher . . . . . School . . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . . .
2	12	22	32 . . . . .
3	13	23	33 . . . . .
4	14	24	34 . . . . .
5	15	25	35 . . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

Tear Here

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