The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, August 13, 2002 — 8:30 to 11:30 a.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

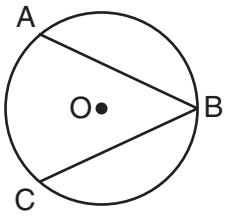
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 Express $\frac{5\pi}{12}$ radians in degrees.

2 What is the image of point $A(1,3)$ after a dilation with the center at the origin and a scale factor of 4?

3 In the accompanying diagram of circle O, $\text{m}\overset{\frown}{ABC} = 260$. What is $\text{m}\angle ABC$?



4 Solve for x: $4^{2x} = 2^{(6x-8)}$

5 If 0.0435 is written as 4.35×10^n, what is the value of n?

6 If $f(x) = x^{-2} + 27^x$, find $f\left(\frac{2}{3}\right)$ in simplest form.

7 Express $\sqrt{-27} + 7\sqrt{-12}$ as a monomial in terms of i.

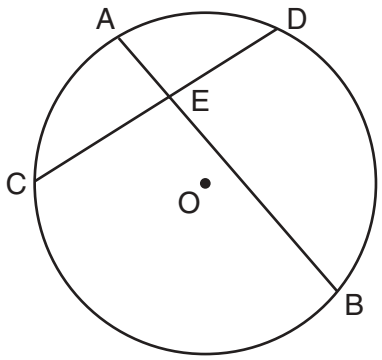
8 If $f(x) = \frac{x^3}{3}$ and $g(x) = \sqrt[3]{x}$, find $f(g(9))$.

9 Evaluate: $\sum_{k=3}^{6} \frac{1}{2}k^2$

10 Find the sum of the roots of the equation $x^2 + 7x - 8 = 0$.

11 In $\triangle RST$, $\sin T = \frac{1}{5}$, $\text{m}\angle R = 30$, and $r = 15$. What is the length of t?

12 In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E, $AE = 5$, $CD = 18$, and $ED = 8$. Find the length of $\overline{EB}$.



13 Express in simplest form: $\dfrac{x - \frac{1}{x}}{1 + \frac{1}{x}}$

14 What is the greatest value of c for which the roots of the equation $x^2 + 4x + c = 0$ are real?

15 Express $\sin 150°$ as a function of a positive acute angle.

16 In $\triangle DEF$, $\text{m}\angle D = 40$, $DE = 12$ meters, and $DF = 8$ meters. Find the area of $\triangle DEF$ to the *nearest tenth of a square meter.*

Directions (17–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

17 For which value of x is the expression $\dfrac{(1 - \sin x)(1 + \sin x)}{\cos x}$ undefined?

(1) 0°
(2) 45°
(3) 90°
(4) 180°

18 What are the coordinates of P', the image of $P(-1,4)$ after a reflection in the line $x = 2$?

(1) (4,1) (3) (0,4)
(2) (–1,1) (4) (5,4)

19 The students' scores on a standardized test with a normal distribution have a mean of 500 and a standard deviation of 40. What percent of the students scored between 420 and 580?

(1) 47.5% (3) 95%
(2) 68% (4) 99.5%

20 Which trigonometric function is positive in Quadrant IV?

(1) $\sin x$ (3) $\csc x$
(2) $\sec x$ (4) $\cot x$

21 The expression $\frac{\sqrt{x}}{\sqrt{x}-1}$ is equivalent to

(1) $x + \sqrt{x}$ (3) $\frac{\sqrt{x}-1}{x}$
(2) $\frac{x+\sqrt{x}}{x-1}$ (4) $1 - \sqrt{x}$

22 What is the solution set of the equation $|x - 6| + 4 = 10$?

(1) {0,12} (3) {–12,0}
(2) {–8,12} (4) {–12,–8}

23 What is the period of the graph of the equation $y = 2 \sin 3x$?

(1) $\frac{2\pi}{3}$ (3) 3
(2) 2 (4) π

24 What is the value of x in the equation $x = 2 \text{ Arc sin } \frac{1}{2}$?

(1) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$
(2) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

25 The expression $\sin 2A - 2 \sin A$ is equivalent to

(1) $(\sin A)(\sin A - 2)$ (3) $(\sin A)(2 \cos A - 1)$
(2) $(2 \sin A)(\sin A - 1)$ (4) $(2 \sin A)(\cos A - 1)$

26 The expression $2 \log x - 3 \log y$ is equivalent to

(1) $\log \frac{2x}{3y}$ (3) $\log \frac{x^2}{y^3}$
(2) $\log x^2y^3$ (4) $\frac{2}{3} \log \frac{x}{y}$

27 What is the domain of the function $f(x) = \frac{4}{\sqrt{2x-1}}$ over the set of real numbers?

(1) $\left\{x \middle| x = \frac{1}{2}\right\}$ (3) $\left\{x \middle| x < \frac{1}{2}\right\}$
(2) $\left\{x \middle| x \geq \frac{1}{2}\right\}$ (4) $\left\{x \middle| x > \frac{1}{2}\right\}$

28 The solution of $\log_x 8 = 2$ is

(1) $x < 2$ (3) $3 < x < 4$
(2) $2 < x < 3$ (4) $x > 4$

29 Which statement is true if r varies inversely as s?

(1) Their difference will be constant.
(2) Their sum will be constant.
(3) Their quotient will be constant.
(4) Their product will be constant.

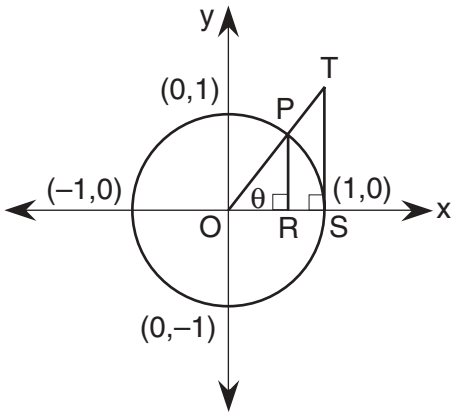
30 Which two values of x satisfy the equation $\sqrt{3 - 2 \cos x} = 2$?

(1) 150° and 210° (3) 60° and 300°
(2) 120° and 240° (4) 30° and 330°

31 Which statement best describes a triangle that can be constructed if $m\angle A = 30$, $a = \frac{1}{4}$, and $b = \frac{1}{2}$?

(1) It is a right triangle.
(2) It is an obtuse triangle.
(3) It is not unique.
(4) It cannot be constructed.

32 In the accompanying diagram, $\overline{TS}$ is tangent to unit circle O at S, $\overline{PR} \perp \overline{OS}$, and $\overline{TS} \perp \overline{OS}$.



Which line segment represents $\sin \theta$?

(1) $\overline{OR}$ (3) $\overline{TS}$

(2) $\overline{OS}$ (4) $\overline{PR}$

33 What is the solution set of the inequality $x^2 - x > 20$?

(1) $\{x > 5\}$ (3) $\{x > 5 \text{ or } x < -4\}$

(2) $\{-4 < x < 5\}$ (4) $\{x > 0\}$

34 What is the solution set of the equation $x^2 + 9 = 0$?

(1) $\{3,-3\}$ (3) $\{-3,-3i\}$

(2) $\{3i,-3i\}$ (4) $\{\ \}$

35 The third term in the expansion of $(x - 2y)^6$ is

(1) $60x^4y^2$ (3) $15x^4y^2$

(2) $60x^2y^4$ (4) $-15x^2y^4$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations $y = -4 \cos x$ and $y = \tan x$ in the interval $-\pi \leq x \leq \pi$. [8]

b Using the graphs sketched in part *a*, determine the number of values of x in the interval $-\pi \leq x \leq \pi$ that satisfy the equation $-4 \cos x = \tan x$. [2]

37 *a* On graph paper, sketch and label the graph of the equation $y = -2^x$. [4]

b On the same set of axes, reflect the graph drawn in part *a* in the line $y = -x$ and label it b. [4]

c Write an equation of the graph drawn in part b. [2]

38 *a* Express in simplest form:

$$\frac{4x^2 - 100}{x^2 + x - 6} \div \frac{20 - 4x}{2x^2 - 9x + 10}$$ [5]

b Express the roots of the equation $-6x = 2x^2 + 5$ in simplest $a + bi$ form. [5]

39 *a* Two forces of 80 pounds and 100 pounds yield a resultant force of 60 pounds. Find, to the *nearest ten minutes* or the *nearest tenth of a degree*, the angle between the two forces. [6]

b Prove the following identity:

$$\frac{(\sin x + \cos x)^2 - 1}{\cos x} = (\sin 2x)(\tan x)(\csc x)$$ [4]

40 A factory that produces light bulbs determined that $\frac{1}{10}$ of all light bulbs it produces are defective.

a If four light bulbs are selected at random, what is the probability that

(1) no bulb selected is defective [2]

(2) *at least* three bulbs selected are defective [3]

b The table below shows the number of defective light bulbs that were found in 20 random samples of 40 light bulbs.

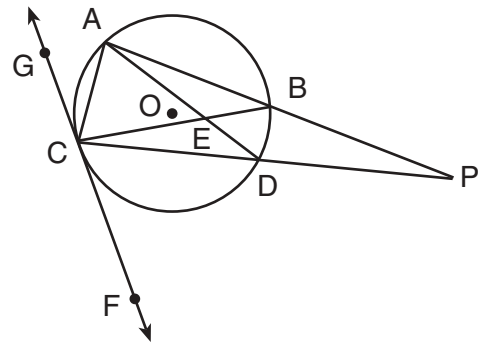
x_i	f_i
0	2
1	2
2	2
3	2
4	4
5	2
6	2
7	2
8	1
9	0
10	1

(1) Find the standard deviation of this set of numbers to the *nearest tenth*. [4]

(2) How many samples fell within one standard deviation of the mean? [1]

41 Find, to the *nearest ten minutes* or the *nearest tenth of a degree*, all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $4\cos^2\theta = 3 + 3\sin\theta$. [10]

42 In the accompanying diagram of circle O, secant $\overline{ABP}$, secant $\overline{CDP}$, and chord $\overline{AC}$ are drawn; chords $\overline{AD}$ and $\overline{BC}$ intersect at E, tangent $\overleftrightarrow{GCF}$ intersects circle O at C, and $\text{m}\overset{\frown}{AB}:\text{m}\overset{\frown}{BD}:\text{m}\overset{\frown}{DC}:\text{m}\overset{\frown}{CA} = 8:2:5:3$.



Find:

a $\text{m}\overset{\frown}{CA}$ [2]

b $\text{m}\angle ACB$ [2]

c $\text{m}\angle P$ [2]

d $\text{m}\angle AEB$ [2]

e $\text{m}\angle DCF$ [2]

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SEQUENTIAL MATH – COURSE III

Tuesday, August 13, 2002 — 8:30 to 11:30 a.m., only

Part I Score	. . . . .
Part II Score	. . . . .
Total Score	. . . . .
Rater's Initials:	. . . . .

ANSWER SHEET

Student . . . . . Sex: ☐ Male ☐ Female Grade . . . . .

Teacher . . . . . School . . . . .

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 . . . . .
2	12	22	32 . . . . .
3	13	23	33 . . . . .
4	14	24	34 . . . . .
5	15	25	35 . . . . .
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

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Tear Here

Tear Here