The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, August 12, 1999 — 8:30 to 11:30 a.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 If $f(x) = x^0 + x^{\frac{1}{2}} + x^{-1}$, find f(4).

2 If $f(x) = \sin x + \cos x$, evaluate $f(2\pi)$.

3 Express the sum of $2\sqrt{-9}$ and $7\sqrt{-64}$ in simplest form in terms of i.

4 Express 220° in radian measure.

5 Evaluate: $\sum_{x=2}^{5}(x-3)^2$

6 For which *negative* value of x is the fraction $\frac{x+5}{x^2-x-6}$ undefined?

7 If $f(x) = x^2$ and $g(x) = 2x - 1$, find $(f \circ g)(4)$.

8 Point A is rotated 180° in a counterclockwise direction about the origin. If the coordinates of A are (–1,3), what are the coordinates of A', its image?

9 Solve for x: $\log_2 x = 3$

10 In which quadrant does the sum of $-4 + 2i$ and $5 - 6i$ lie?

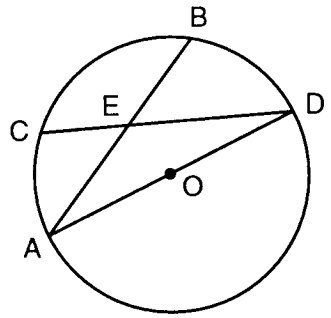
11 In $\triangle ABC$, $\sin A = \frac{1}{3}$, $m\angle B = 30$, and $a = 12$. What is the length of b?

12 Tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn to a circle from external point P. If $PA = 9$ and $PC = 27$, find PB.

13 Solve for x: $32^x = 4^{(2x+1)}$

14 What is the number of degrees in the value of θ that satisfies the equation $2\cos\theta - 1 = 0$ in the interval $180° \le \theta \le 360°$?

15 In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E and $\overline{AD}$ is a diameter. If $m\overset{\frown}{CB} = 82$, find $m\angle AED$.



16 Factor completely: $\tan^3 x - 9\tan x$

Directions (17–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

17 In $\triangle ABC$, $b = 2$, $c = 4$, and $m\angle A = 30$. The area of $\triangle ABC$ is

(1) 1 (3) $\sqrt{3}$
(2) 2 (4) 4

18 Which value of c would make the roots of the equation $x^2 + 6x + c = 0$ real, rational, and equal?

(1) 9 (3) 18
(2) –9 (4) –18

19 Expressed as a single fraction, $\frac{5}{x-3} - \frac{1}{x}$ is equivalent to

(1) $\frac{6x-3}{x^2-3x}$ (3) $\frac{4x+3}{2x-3}$
(2) $\frac{4x+3}{x^2-3x}$ (4) $\frac{4}{x^2-3x}$

20 If x varies inversely as y and y is doubled, then x will be

(1) divided by 2
(2) multiplied by 2
(3) decreased by 2
(4) increased by 2

21 If $\tan x = -\sqrt{3}$, in which quadrants could angle x terminate?

(1) I and III
(2) II and III
(3) II and IV
(4) III and IV

22 If A and B are positive acute angles, $\sin A = \frac{5}{13}$, and $\cos B = \frac{4}{5}$, what is the value of $\sin (A + B)$?

(1) $-\frac{16}{65}$
(2) $\frac{33}{65}$
(3) $\frac{56}{65}$
(4) $\frac{63}{65}$

23 What is the solution set for the equation $2x + |x| = -2$?

(1) {1}
(2) {–2}
(3) {–1}
(4) { }

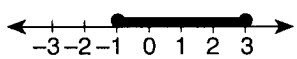
24 The expression $\frac{\sec \theta}{\csc \theta}$ is equivalent to

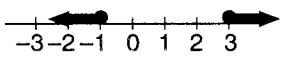
(1) $\cot \theta$
(2) $\tan \theta$
(3) $\cos \theta$
(4) $\sin \theta$

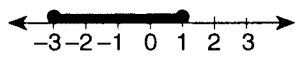
25 On a standardized examination, Laura received a score of 85, which was exactly 2 standard deviations above the mean. If the standard deviation for the examination is 4, what is the mean for this examination?

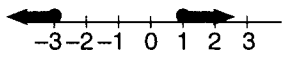
(1) 93
(2) 87
(3) 83
(4) 77

26 What is the solution set for the inequality $x^2 - 2x - 3 \leq 0$?

(1) 

(2) 

(3) 

(4) 

27 The domain of $f(x) = x^2 + 2x + 1$ is $-3 \leq x \leq 3$. The largest value in the range of $f(x)$ is

(1) 20
(2) 16
(3) 3
(4) 4

28 In $\triangle DEF$ if $d = \sqrt{3}$, $e = 4$, and $m\angle F = 30$, the length of f is

(1) 7
(2) $\sqrt{17}$
(3) $\sqrt{7}$
(4) $\sqrt{3}$

29 The maximum value of $3 \sin \frac{1}{3}\theta$ is

(1) 1
(2) $\frac{1}{3}$
(3) 3
(4) 0

30 The graph of the equation $y = 2 \cos 2x$, $0 \leq x \leq 2\pi$, has a line of symmetry at

(1) $x = \pi$
(2) $x = \frac{\pi}{4}$
(3) $y = 2$
(4) the x-axis

31 For the equation $\sqrt{x + 21} = x + 1$, the solution set for x is

(1) { }
(2) {–5}
(3) {–5,4}
(4) {4}

32 If $\sin x = -\frac{\sqrt{2}}{2}$ and $\cos x = \frac{\sqrt{2}}{2}$, the measure of angle x is

(1) 45°
(2) 135°
(3) 225°
(4) 315°

33 The graph of the equation $y = -(4)^x$ lies in Quadrants

(1) I and II
(2) II and III
(3) III and IV
(4) I and IV

34 Which quadratic equation has roots of $3 - i$ and $3 + i$?

(1) $x^2 + 6x + 10 = 0$
(2) $x^2 + 6x + 8 = 0$
(3) $x^2 - 6x + 8 = 0$
(4) $x^2 - 6x + 10 = 0$

35 The graph of the equation $y^2 - x^2 = 4$ forms

(1) a circle
(2) an ellipse
(3) a hyperbola
(4) a parabola

Answers to the following questions are to be written on paper provided by the school.

Part II

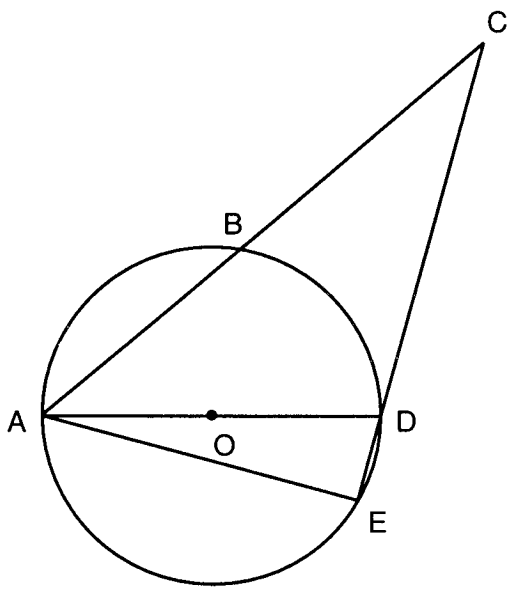
Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On graph paper, sketch and label the graph of the equation $y = 2\cos\theta$ in the interval $0 \le \theta \le 2\pi$. [4]

b On the same set of axes, sketch and label the graph of the equation $y = \tan\theta$ in the same interval. [4]

c If $f(\theta) = \tan\theta - 2\cos\theta$, find $f(\pi)$. [2]

37 In the accompanying diagram of circle O, diameter $\overline{AD}$, chord $\overline{AE}$, and secants $\overline{CBA}$ and $\overline{CDE}$ are drawn; $m\angle BAD = 40$; and $m\overset{\frown}{AE} = 5(m\overset{\frown}{ED})$.



Find:

a $m\overset{\frown}{BD}$ [2]

b $m\overset{\frown}{AE}$ [2]

c $m\angle ACE$ [2]

d $m\angle AED$ [2]

e $m\angle ADC$ [2]

38 *a* On graph paper, draw and label $\triangle PQR$, whose vertices are $P(3,5)$, $Q(9,5)$, and $R(7,7)$. [1]

b On the same set of axes, graph and state the coordinates of

(1) $\triangle P'Q'R'$, the image of $\triangle PQR$ after $R_{90°}$ [3]

(2) $\triangle P''Q''R''$, the image of $\triangle P'Q'R'$ after $r_{x\text{-axis}}$ [2]

(3) $\triangle P'''Q'''R'''$, the image of $\triangle P''Q''R''$ after $r_{y\text{-axis}}$ [2]

c Based on the graphs drawn in parts *a* and *b*, write a single transformation that shows the composition $r_{y\text{-axis}} \circ r_{x\text{-axis}} \circ R_{90°}$. [2]

39 *a* Perform the indicated operations and express in lowest terms:

$$\frac{x^2-9}{2x+4} \cdot \frac{x^2+7x+10}{x^2-3x-18} \div \frac{x^2+2x-15}{2x^2-12x} \quad [6]$$

b Express the roots of the equation $2x^2 + 4x + 5 = 0$ in simplest $a + bi$ form. [4]

40 *a* Prove that the following is an identity for all values of θ for which the expressions are defined:

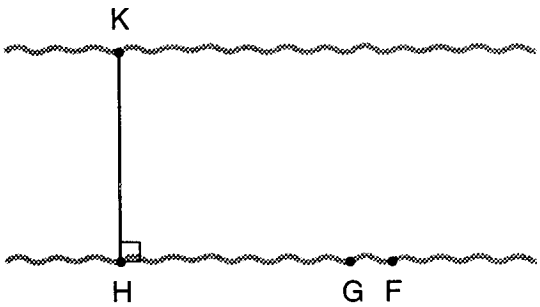
$$\frac{\sin\theta}{\cot\theta} + \cos\theta = \sec\theta \quad [4]$$

b If θ is in Quadrant II and $\cos\theta = -\frac{3}{4}$, find an exact value for $\sin 2\theta$. [3]

c Solve for x to the *nearest hundredth*:

$$\log_7 75 = x \quad [3]$$

41 To determine the distance across a river, a surveyor marked three points on one riverbank: H, G, and F, as shown below. She also marked one point, K, on the opposite bank such that $\overline{KH} \perp \overline{HGF}$, $m\angle KGH = 41$, and $m\angle KFH = 37$. The distance between G and F is 45 meters. Find KH, the width of the river, to the *nearest tenth of a meter*. [10]



42 *a* A spinner is divided into two regions, green and red. The probability of the pointer landing on the green region is $\frac{2}{3}$. The pointer is spun 5 times.

(1) What is the probability of the pointer landing on the green region *exactly* 2 times? [2]

(2) What is the probability of the pointer landing on the red region *at least* 4 times? [3]

b Find all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $\cos \theta = \cos 2\theta$. [5]

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Thursday, August 12, 1999 — 8:30 to 11:30 a.m., only

Part I Score	.....
Part II Score	.....
Total Score	.....
Rater's Initials:	.....

ANSWER SHEET

Pupil ..... Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

1	11	21	31 .....
2	12	22	32 .....
3	13	23	33 .....
4	14	24	34 .....
5	15	25	35 .....
6	16	26	
7	17	27	
8	18	28	
9	19	29	
10	20	30	

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Tear Here

Signature

Tear Here

Tear Here