

# GEOMETRY

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

Tuesday, January 26, 2016 — 1:15 to 4:15 p.m., only

Student Name: \_\_\_\_\_

School Name: \_\_\_\_\_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

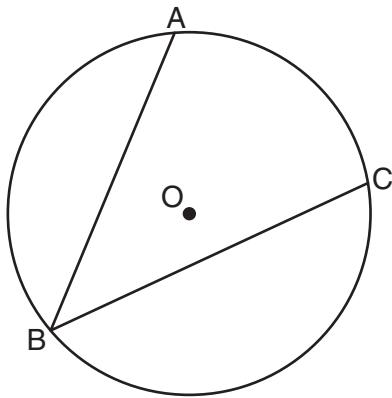
**Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.** [56]

**Use this space for computations.**

- 1** What is the equation of a circle with its center at  $(5, -2)$  and a radius of 3?

- (1)  $(x - 5)^2 + (y + 2)^2 = 3$
- (2)  $(x - 5)^2 + (y + 2)^2 = 9$
- (3)  $(x + 5)^2 + (y - 2)^2 = 3$
- (4)  $(x + 5)^2 + (y - 2)^2 = 9$

- 2** In the diagram below,  $\angle ABC$  is inscribed in circle  $O$ .

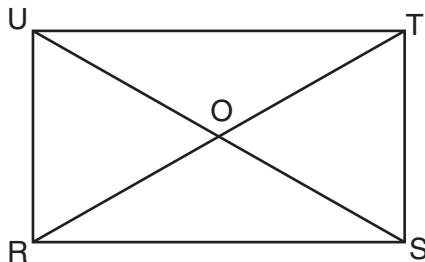


The ratio of the measure of  $\angle ABC$  to the measure of  $\widehat{AC}$  is

- (1) 1:1
- (2) 1:2
- (3) 1:3
- (4) 1:4

**Use this space for computations.**

- 3 In the diagram below of rectangle  $RSTU$ , diagonals  $\overline{RT}$  and  $\overline{SU}$  intersect at  $O$ .

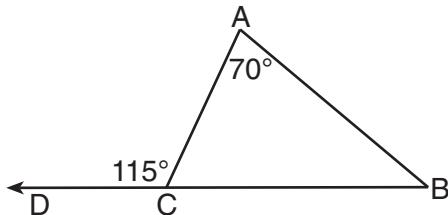


If  $RT = 6x + 4$  and  $SO = 7x - 6$ , what is the length of  $\overline{US}$ ?

- |       |        |
|-------|--------|
| (1) 8 | (3) 16 |
| (2) 2 | (4) 32 |
- 4 How many points are 3 units from the origin and also equidistant from both the  $x$ -axis and  $y$ -axis?
- |       |       |
|-------|-------|
| (1) 1 | (3) 0 |
| (2) 2 | (4) 4 |
- 5 The converse of the statement “If a triangle has one right angle, the triangle has two acute angles” is
- (1) If a triangle has two acute angles, the triangle has one right angle.
  - (2) If a triangle has one right angle, the triangle does not have two acute angles.
  - (3) If a triangle does not have one right angle, the triangle does not have two acute angles.
  - (4) If a triangle does not have two acute angles, the triangle does not have one right angle.
- 6 The surface area of a sphere is  $2304\pi$  square inches. The length of a radius of the sphere, in inches, is
- |        |         |
|--------|---------|
| (1) 12 | (3) 288 |
| (2) 24 | (4) 576 |

**Use this space for computations.**

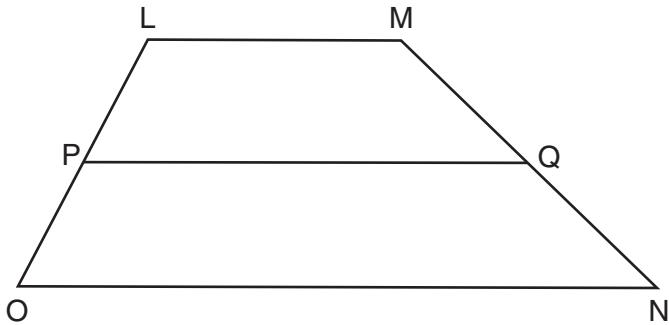
- 7 As shown in the diagram below of  $\triangle ABC$ ,  $\overline{BC}$  is extended through  $D$ ,  $m\angle A = 70^\circ$ , and  $m\angle ACD = 115^\circ$ .



Which statement is true?

- |               |               |
|---------------|---------------|
| (1) $AC > AB$ | (3) $BC < AC$ |
| (2) $AB > BC$ | (4) $AC < AB$ |

- 8 In trapezoid  $LMNO$  below, median  $\overline{PQ}$  is drawn.



If  $LM = x + 7$ ,  $ON = 3x + 11$ , and  $PQ = 25$ , what is the value of  $x$ ?

- |          |        |
|----------|--------|
| (1) 1.75 | (3) 8  |
| (2) 3.5  | (4) 17 |

- 9 Points  $A$  and  $B$  are on line  $\ell$ . How many points are 3 units from line  $\ell$  and also equidistant from  $A$  and  $B$ ?

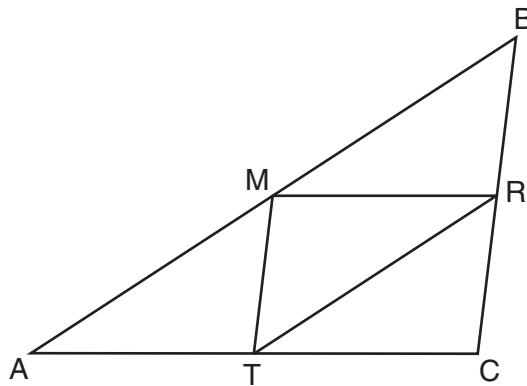
- |       |       |
|-------|-------|
| (1) 1 | (3) 3 |
| (2) 2 | (4) 4 |

**Use this space for computations.**

- 10** The lines whose equations are  $2x + 3y = 4$  and  $y = mx + 6$  will be perpendicular when  $m$  is

- (1)  $-\frac{3}{2}$     (3)  $\frac{3}{2}$   
(2)  $-\frac{2}{3}$     (4)  $\frac{2}{3}$

- 11** As shown in the diagram below,  $M$ ,  $R$ , and  $T$  are midpoints of the sides of  $\triangle ABC$ .

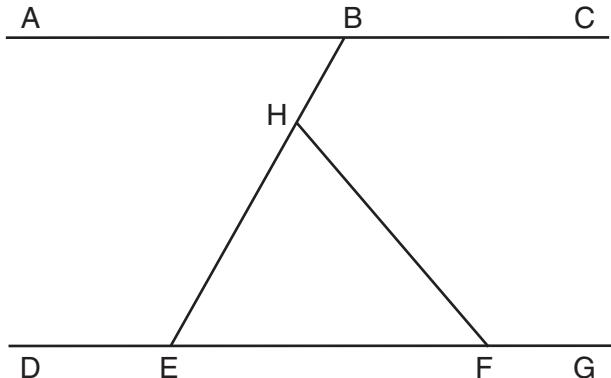


If  $AB = 18$ ,  $AC = 14$ , and  $BC = 10$ , what is the perimeter of quadrilateral  $ACRM$ ?

- (1) 35    (3) 24  
(2) 32    (4) 21

**Use this space for computations.**

- 12 In the diagram below,  $\overline{ABC} \parallel \overline{DEFG}$ . Transversal  $\overline{BHE}$  and line segment  $HF$  are drawn.



If  $m\angle HFG = 130$  and  $m\angle EHF = 70$ , what is  $m\angle ABE$ ?

- (1) 40
- (3) 60
- (2) 50
- (4) 70

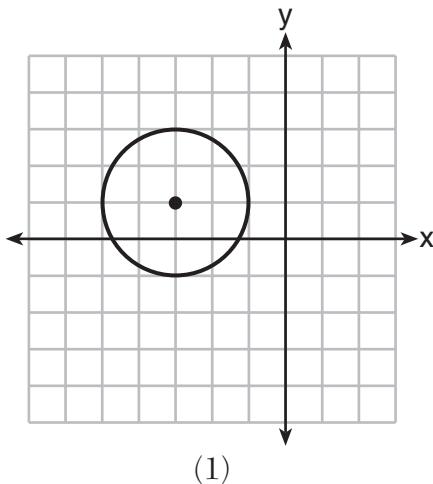
- 13 The graphs of the lines represented by the equations

$$y = \frac{1}{3}x + 7 \text{ and } y = -\frac{1}{3}x - 2 \text{ are}$$

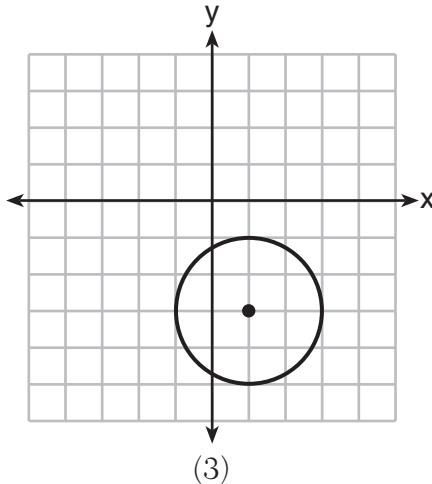
- (1) parallel
- (2) horizontal
- (3) perpendicular
- (4) intersecting, but not perpendicular

**Use this space for computations.**

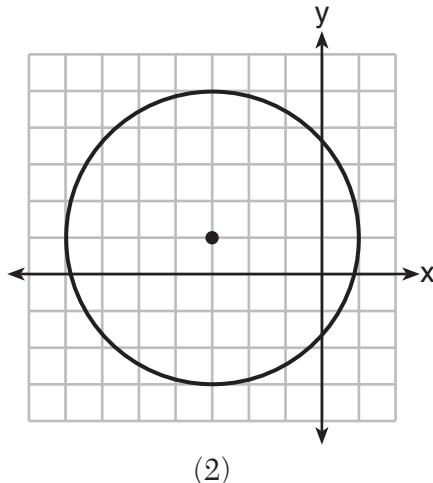
- 14 Which graph represents a circle whose equation is  
 $(x + 3)^2 + (y - 1)^2 = 4$ ?



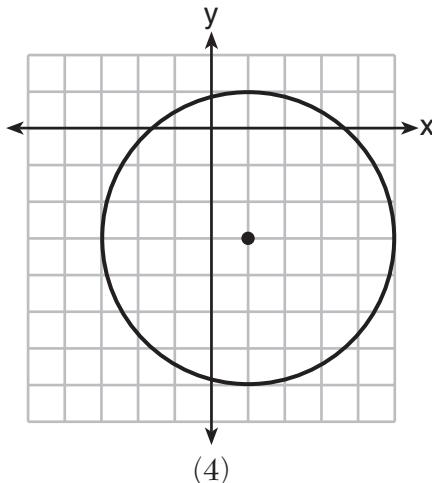
(1)



(3)



(2)



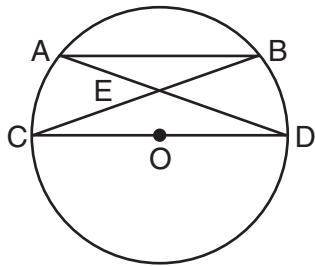
(4)

- 15 In  $\triangle ABC$ ,  $m\angle CAB = 2x$  and  $m\angle ACB = x + 30$ . If  $\overline{AB}$  is extended through point  $B$  to point  $D$ ,  $m\angle CBD = 5x - 50$ . What is the value of  $x$ ?

- (1) 25                                  (3) 40  
(2) 30                                    (4) 46

**Use this space for computations.**

- 16** In circle  $O$  shown below, chord  $\overline{AB}$  and diameter  $\overline{CD}$  are parallel, and chords  $\overline{AD}$  and  $\overline{BC}$  intersect at point  $E$ .



Which statement is *false*?

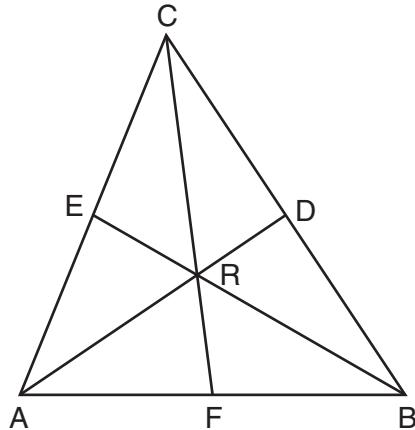
- |                                       |  |
|---------------------------------------|--|
| (1) $\widehat{AC} \cong \widehat{BD}$ | (3) $\triangle ABE \sim \triangle CDE$ |
| (2) $BE = CE$                         | (4) $\angle B \cong \angle C$          |
- 17** When the transformation  $T_{2,-1}$  is performed on point  $A$ , its image is point  $A'(-3,4)$ . What are the coordinates of  $A$ ?
- |              |               |
|--------------|---------------|
| (1) $(5,-5)$ | (3) $(-1,3)$  |
| (2) $(-5,5)$ | (4) $(-6,-4)$ |

- 18** If the sum of the interior angles of a polygon is  $1440^\circ$ , then the polygon must be

- |                |               |
|----------------|---------------|
| (1) an octagon | (3) a hexagon |
| (2) a decagon  | (4) a nonagon |

**Use this space for computations.**

- 19 In  $\triangle ABC$  shown below, medians  $\overline{AD}$ ,  $\overline{BE}$ , and  $\overline{CF}$  intersect at point  $R$ .



If  $CR = 24$  and  $RF = 2x - 6$ , what is the value of  $x$ ?

- |        |        |
|--------|--------|
| (1) 9  | (3) 15 |
| (2) 12 | (4) 27 |

- 20 Which equation represents a line that passes through the point  $(-2,6)$  and is parallel to the line whose equation is  $3x - 4y = 6$ ?

- |                    |                     |
|--------------------|---------------------|
| (1) $3x + 4y = 18$ | (3) $-3x + 4y = 30$ |
| (2) $4x + 3y = 10$ | (4) $-4x + 3y = 26$ |

- 21 The bases of a right prism are triangles in which  $\triangle MNP \cong \triangle RST$ . If  $MP = 9$ ,  $MR = 18$ , and  $MN = 12$ , what is the length of  $\overline{NS}$ ?

- |        |        |
|--------|--------|
| (1) 9  | (3) 15 |
| (2) 12 | (4) 18 |

**Use this space for computations.**

**22** Triangle  $ABC$  has the coordinates  $A(3,0)$ ,  $B(3,8)$ , and  $C(6,6)$ . If  $\triangle ABC$  is reflected over the line  $y = x$ , which statement is true about the image of  $\triangle ABC$ ?

- (1) One point remains fixed.
- (2) The size of the triangle changes.
- (3) The orientation does not change.
- (4) One side of  $\triangle ABC$  is parallel to the line  $y = x$ .

**23** A right circular cone has a diameter of  $10\sqrt{2}$  and a height of 12. What is the volume of the cone in terms of  $\pi$ ?

- (1)  $200\pi$
- (3)  $800\pi$
- (2)  $600\pi$
- (4)  $2400\pi$

**24** Which statement is *not* always true when  $\triangle ABC \cong \triangle XYZ$ ?

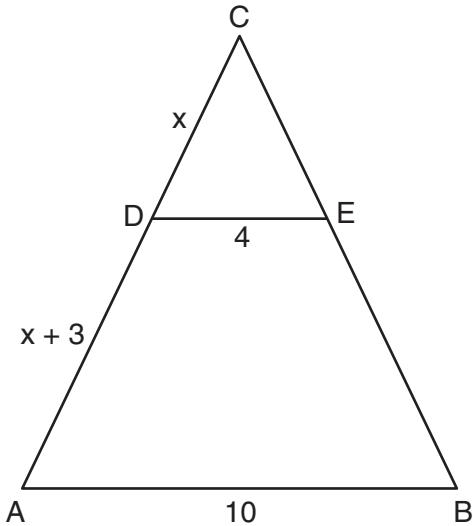
- (1)  $\overline{BC} \cong \overline{YZ}$
- (3)  $\angle CAB \cong \angle ZXY$
- (2)  $\overline{CA} \cong \overline{XY}$
- (4)  $\angle BCA \cong \angle YZX$

**25** If two sides of a triangle have lengths of  $\frac{1}{4}$  and  $\frac{1}{5}$ , which fraction can *not* be the length of the third side?

- (1)  $\frac{1}{9}$
- (3)  $\frac{1}{3}$
- (2)  $\frac{1}{8}$
- (4)  $\frac{1}{2}$

**Use this space for computations.**

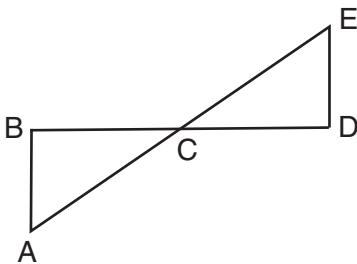
- 26 In the diagram below of  $\triangle ABC$ ,  $\overline{CDA}$ ,  $\overline{CEB}$ ,  $\overline{DE} \parallel \overline{AB}$ ,  $DE = 4$ ,  $AB = 10$ ,  $CD = x$ , and  $DA = x + 3$ .



What is the value of  $x$ ?

- (1) 0.5                          (3) 5.5  
(2) 2                              (4) 6

- 27 Given:  $\overline{AE}$  bisects  $\overline{BD}$  at  $C$   
 $\overline{AB}$  and  $\overline{DE}$  are drawn  
 $\angle ABC \cong \angle EDC$

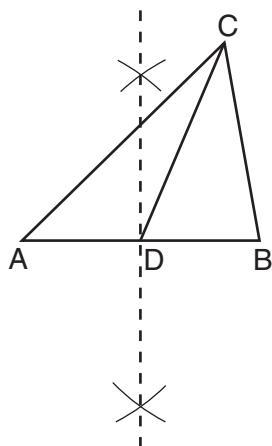


Which statement is needed to prove  $\triangle ABC \cong \triangle EDC$  using ASA?

- (1)  $\angle ABC$  and  $\angle EDC$  are right angles.  
(2)  $\overline{BD}$  bisects  $\overline{AE}$  at  $C$ .  
(3)  $\angle BCA \cong \angle DCE$   
(4)  $\angle DEC \cong \angle BAC$

**Use this space for computations.**

- 28 In the construction shown below,  $\overline{CD}$  is drawn.



In  $\triangle ABC$ ,  $\overline{CD}$  is the

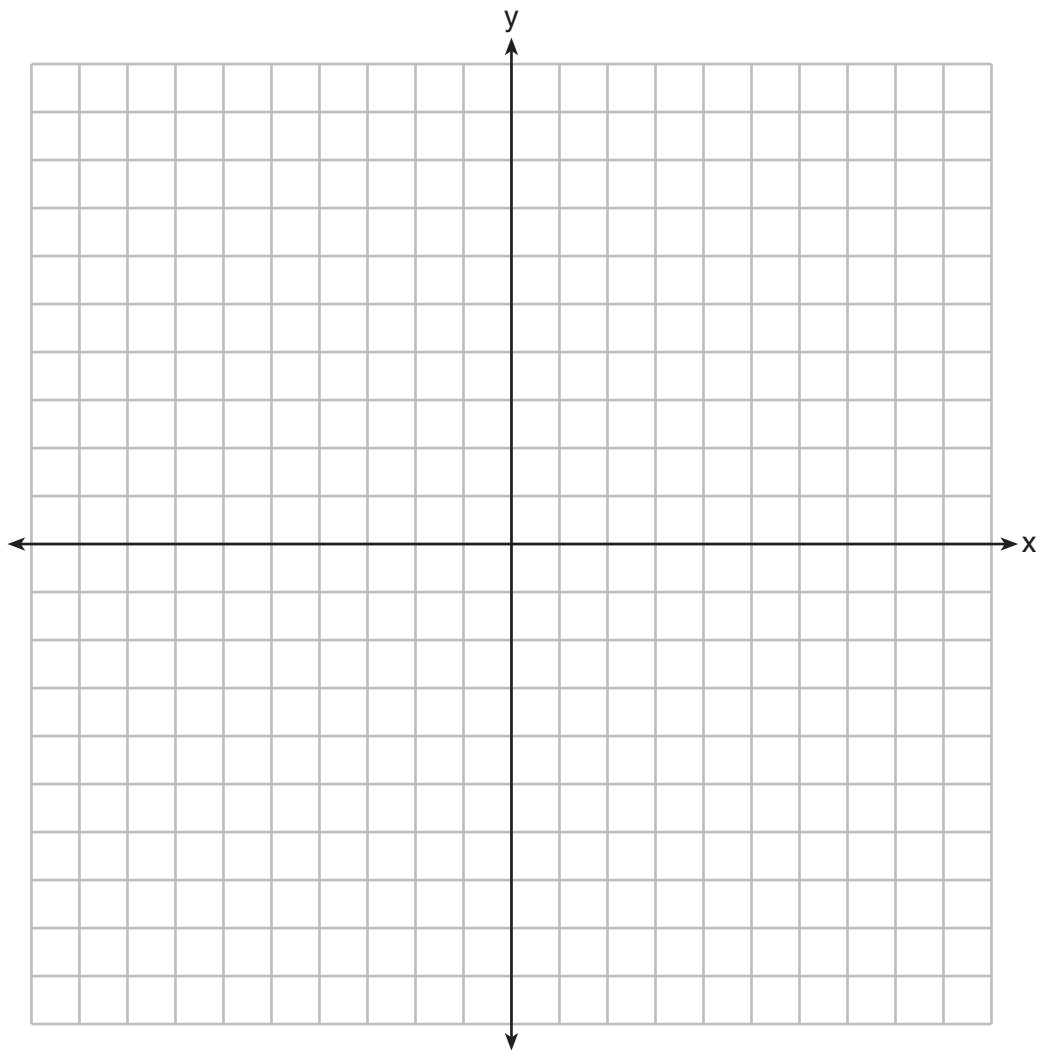
- (1) perpendicular bisector of side  $\overline{AB}$
  - (2) median to side  $\overline{AB}$
  - (3) altitude to side  $\overline{AB}$
  - (4) bisector of  $\angle ACB$
-

## **Part II**

**Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

- 29** The sides of a triangle measure 7, 4, and 9. If the longest side of a similar triangle measures 36, determine and state the length of the shortest side of this triangle.

- 30** Triangle  $ABC$  has coordinates  $A(6, -4)$ ,  $B(0, 2)$ , and  $C(6, 2)$ . On the set of axes below, graph and label  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after a dilation of  $\frac{1}{2}$ .

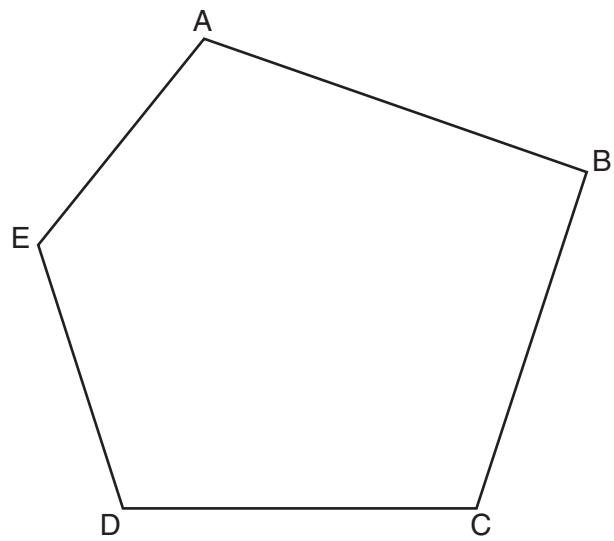


- 31** In parallelogram  $RSTU$ ,  $m\angle R = 5x - 2$  and  $m\angle S = 3x + 10$ .  
Determine and state the value of  $x$ .

**32** Determine and state the length of a line segment whose endpoints are  $(6,4)$  and  $(-9,-4)$ .

- 33** The base of a right pentagonal prism has an area of 20 square inches. If the prism has an altitude of 8 inches, determine and state the volume of the prism, in cubic inches.

- 34** Using a compass and a straightedge, construct the bisector of  $\angle CDE$ .  
[Leave all construction marks.]



### Part III

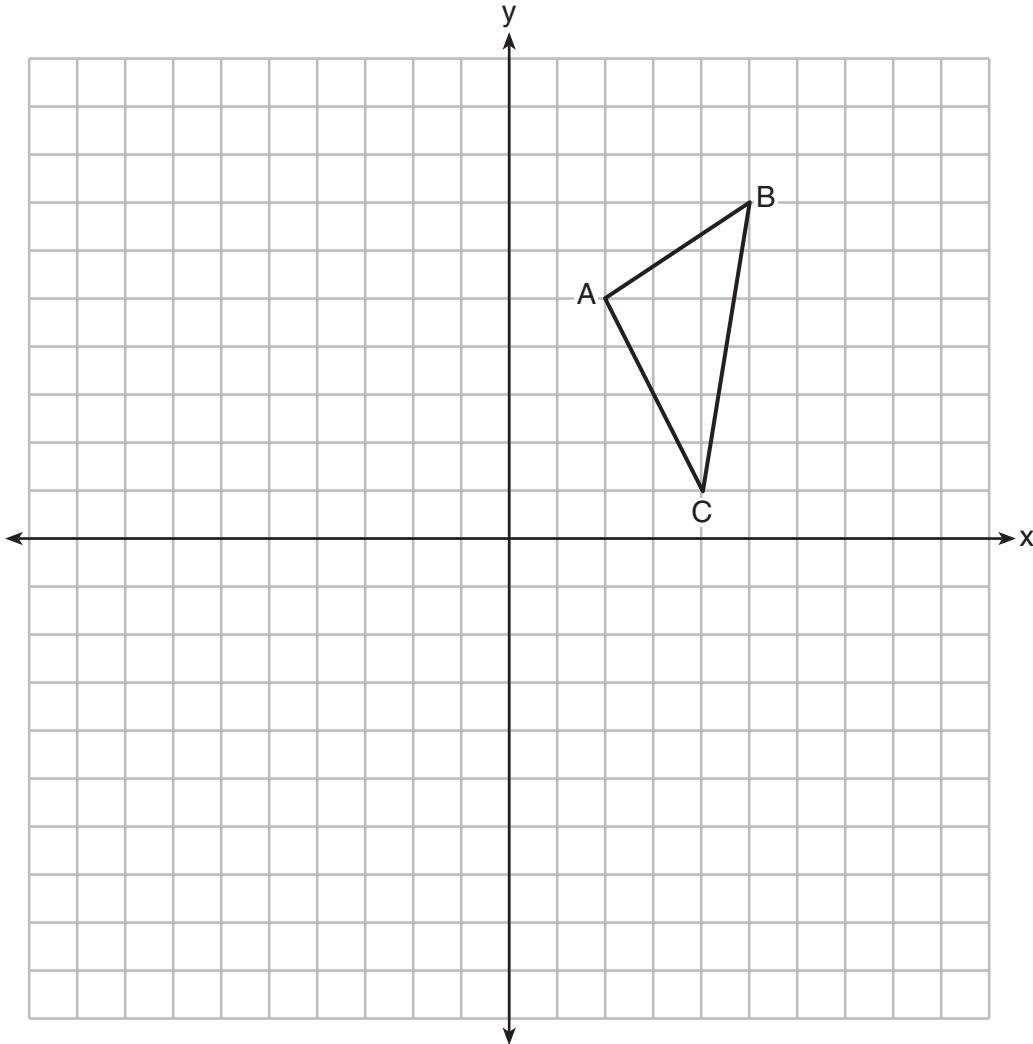
Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

- 35 The coordinates of  $\triangle ABC$ , shown on the graph below, are  $A(2,5)$ ,  $B(5,7)$ , and  $C(4,1)$ .

Graph and label  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after it is reflected over the  $y$ -axis.

Graph and label  $\triangle A''B''C''$ , the image of  $\triangle A'B'C'$  after it is reflected over the  $x$ -axis.

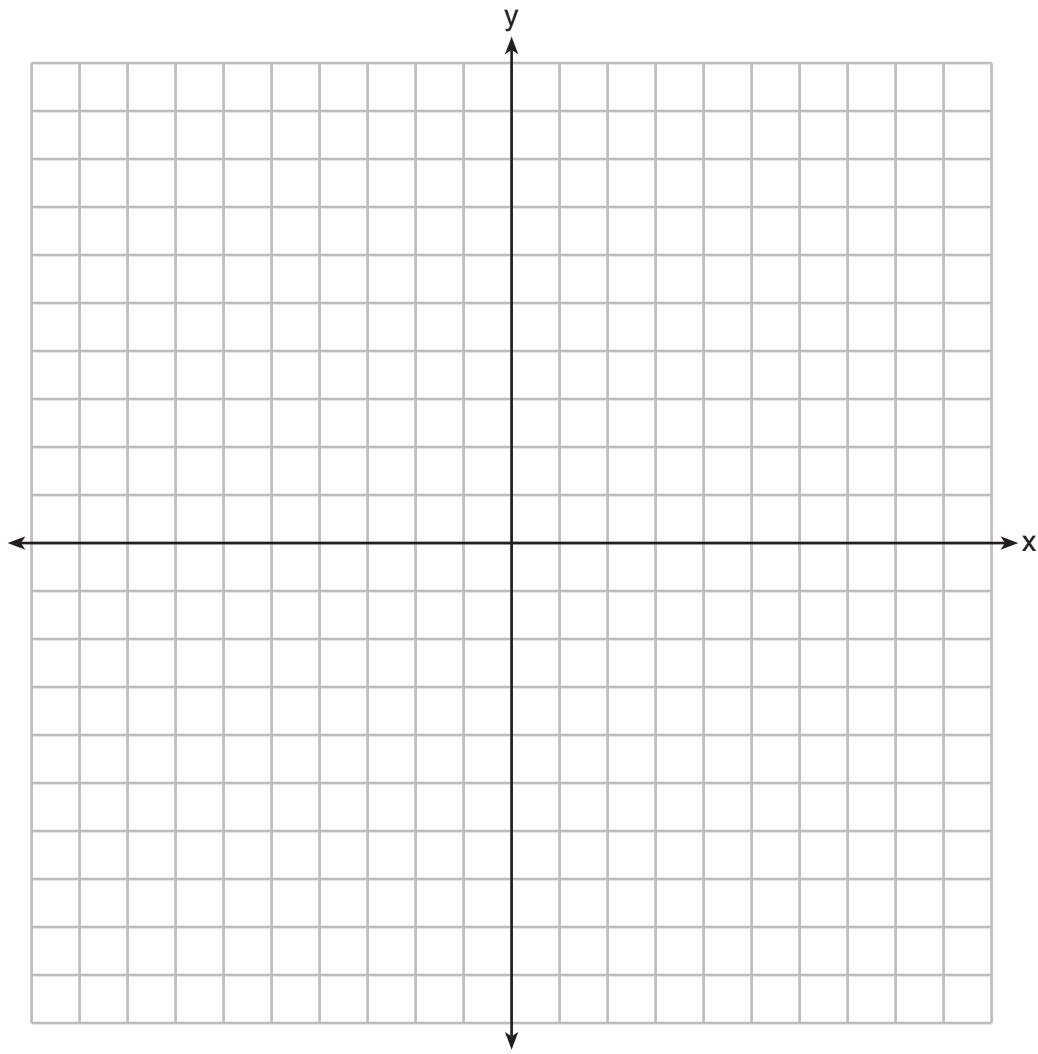
State a single transformation that will map  $\triangle ABC$  onto  $\triangle A''B''C''$ .



**36** On the set of axes below, solve the following system of equations graphically and state the coordinates of all points in the solution.

$$y = x^2 + 4x + 2$$

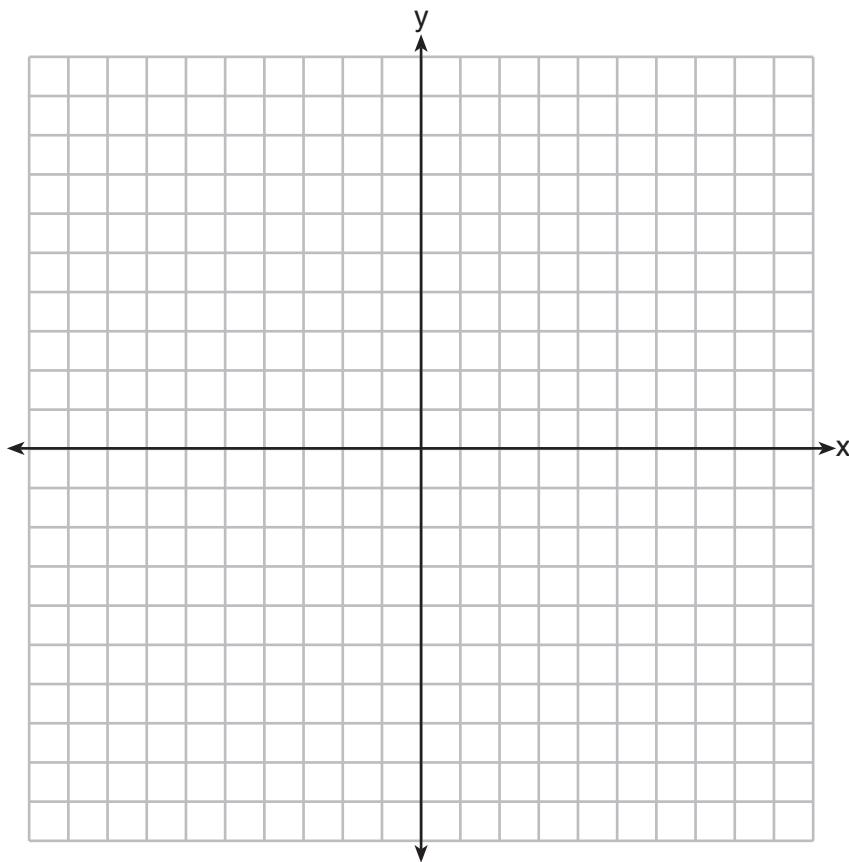
$$y - 2x = 5$$



**37** Given: Triangle  $RST$  has coordinates  $R(-1, 7)$ ,  $S(3, -1)$ , and  $T(9, 2)$

Prove:  $\triangle RST$  is a right triangle

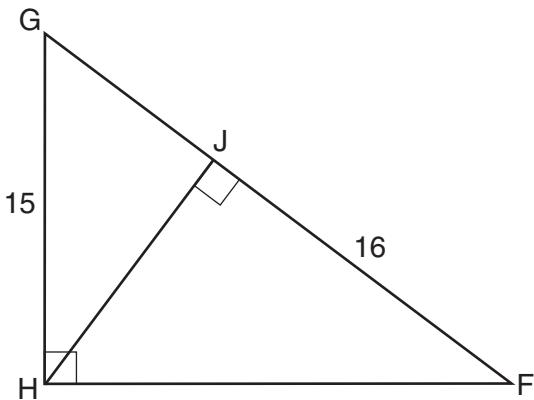
[The use of the set of axes below is optional.]



**Part IV**

**Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen, except for graphs and drawings, which should be done in pencil.** [6]

- 38 In right triangle  $FGH$  shown below,  $m\angle GHF = 90$ , altitude  $\overline{HJ}$  is drawn to  $\overline{FG}$ ,  $FJ = 16$ , and  $HG = 15$ .



Determine and state the length of  $\overline{JG}$ . [Only an algebraic solution can receive full credit.]

Determine and state the length of  $\overline{HJ}$ . [Only an algebraic solution can receive full credit.]

## Reference Sheet

Volume	Cylinder	$V = Bh$ where $B$ is the area of the base
	Pyramid	$V = \frac{1}{3}Bh$ where $B$ is the area of the base
	Right Circular Cone	$V = \frac{1}{3}Bh$ where $B$ is the area of the base
	Sphere	$V = \frac{4}{3}\pi r^3$

Lateral Area ( $L$ )	Right Circular Cylinder	$L = 2\pi rh$
	Right Circular Cone	$L = \pi rl$ where $l$ is the slant height

Surface Area	Sphere	$SA = 4\pi r^2$
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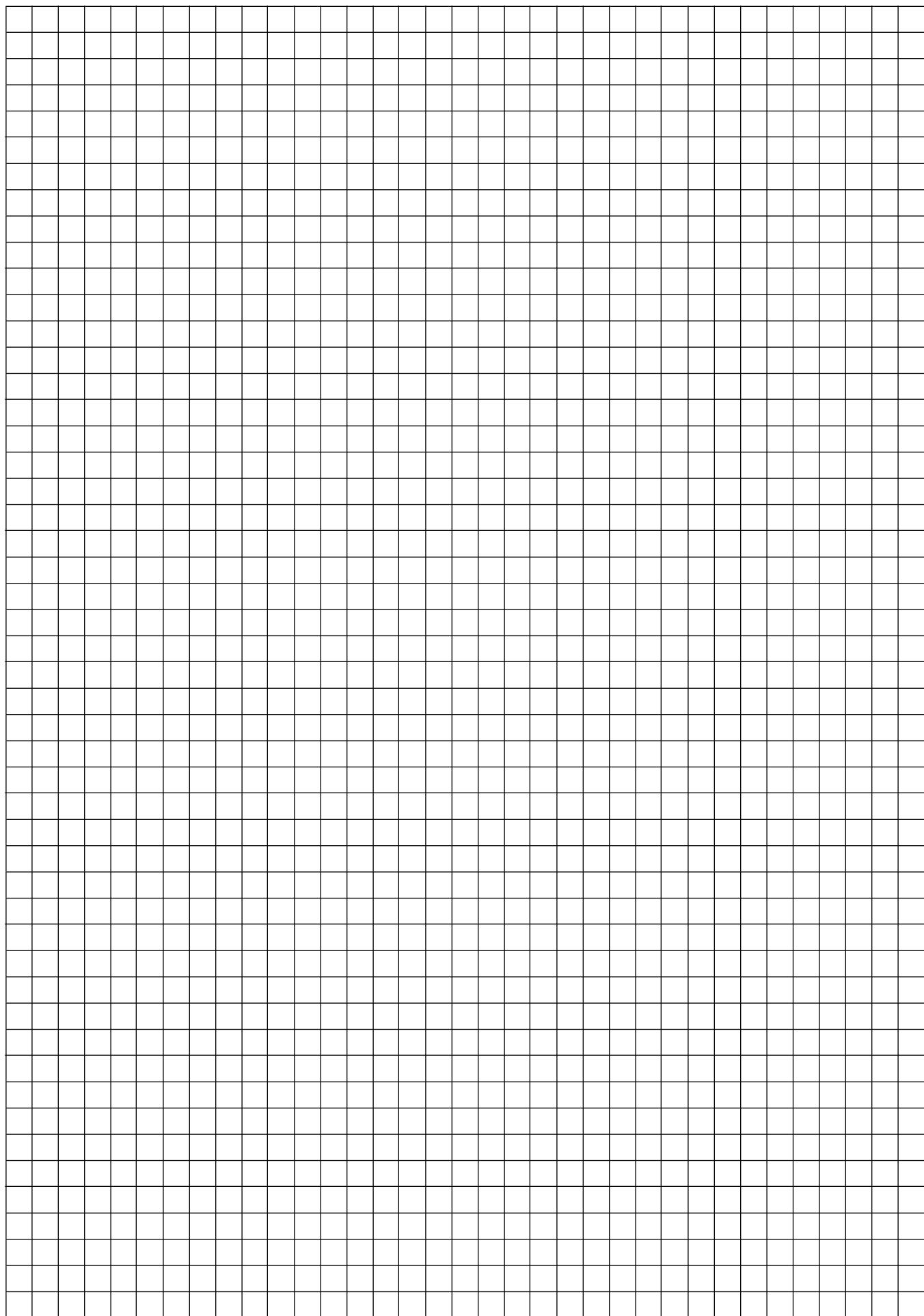
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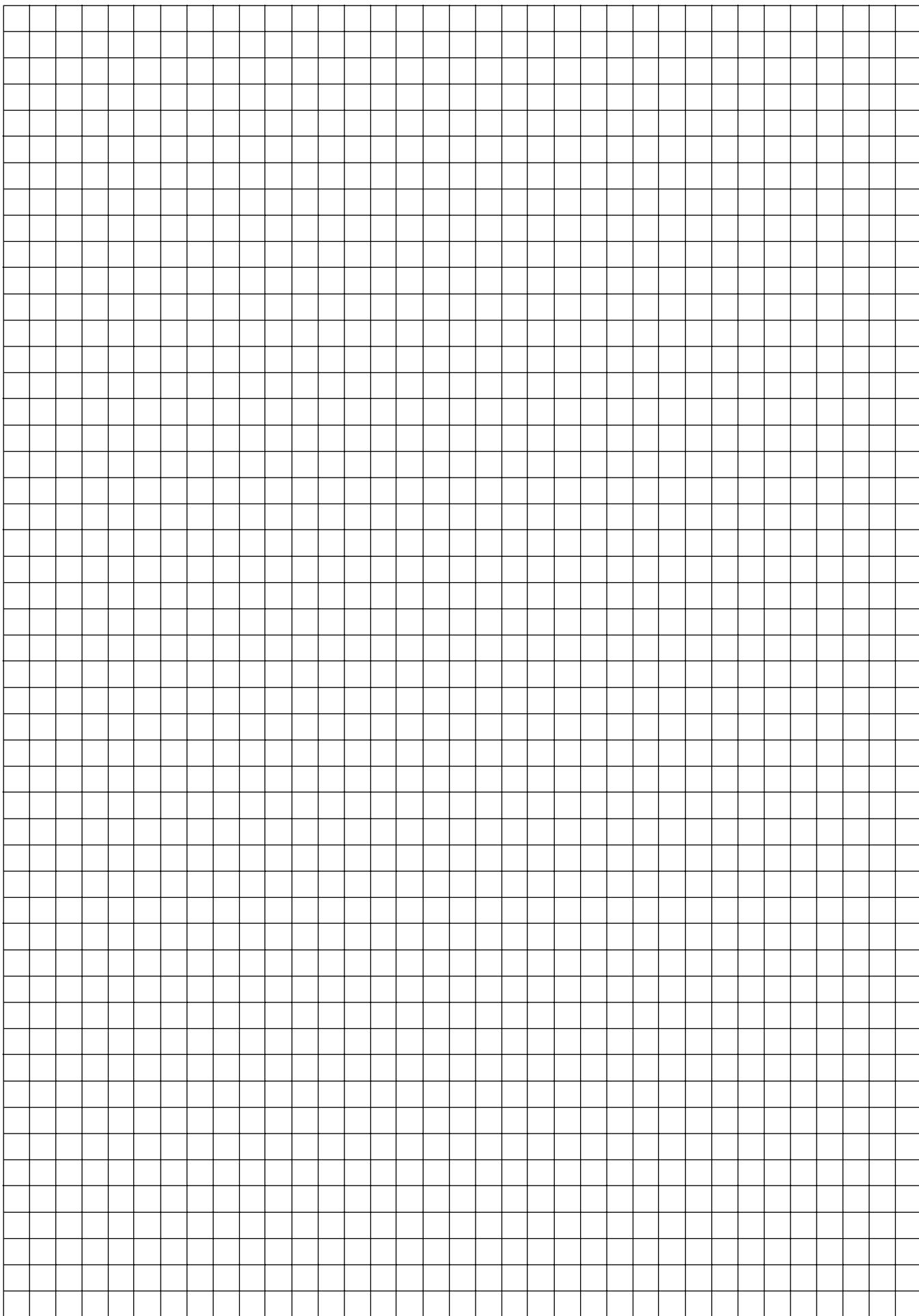
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